1. Local Galaxy Luminosity Function

The local galaxy B-band luminosity function is characterized by the following Schechter parameters: faint-end slope, $\alpha = -1.03$; characteristic absolute magnitude, $M_B^* = -20.04$; and normalization, $\phi^* = 5.90 \times 10^{-3} \text{ Mpc}^{-3}$.

(a) What is the B-band luminosity density, $j_B$, associated with this luminosity function? Express your answer in units of solar luminosity per Mpc$^3$.

(b) What fraction of $j_B$ is contributed by galaxies with $M_B \leq -20.00$?

(c) What is the number-density of galaxies integrated down to $M_B = -20.00$?

(d) Galaxies in the local universe are described by a bimodal distribution in optical color, i.e. they can be divided into samples of “Blue” and “Red” objects. The parameters of the Blue-galaxy luminosity function are: $(\alpha, M_B^*, \phi^*) = (-1.24, -20.09, 2.396 \times 10^{-3})$, while corresponding parameters for the Red-galaxy luminosity function are: $(\alpha, M_B^*, \phi^*) = (-0.76, -19.91, 3.596 \times 10^{-3})$. What fractions of the total B-band luminosity density are contributed by Blue and Red galaxies, respectively? How does this answer change if you only integrate down to $M_B = -20.00$?

2. The Exponential Profile of NGC 5055

NGC 5055 (The Sunflower Galaxy) is an Sbc galaxy at a distance of $d = 9 \text{ Mpc}$. A photograph of the galaxy can be found on-line or in the Hubble Atlas of Galaxies.

(a) From the photograph, estimate the angle of inclination, $i$, between the disk plane and the plane of the sky.

(b) $r$-band photometry along the major axis of NGC 5055 gives $\mu_r = 20.1$ at 1’, $\mu_r = 21.1$ at 2’, $\mu_r = 22.7$ at 4’, and $\mu_r = 25.25$ at 8’ (units of $\mu_r$ are $r$ mags arcsec$^{-2}$). Suppose the stellar disk, as viewed face-on, has an exponential surface-brightness profile, $\Sigma_r = \Sigma_0 \exp(-r/h)$. From your fit to the major axis photometry, calculate $h$, $\Sigma_0$, and the total luminosity of the disk in solar luminosities (the sun has $M_r = 4.74$).
3. Deep into the Fundamental Plane

The Fundamental Plane (FP) of elliptical galaxies is often expressed in the form:

$$\log R_e = \alpha \log \sigma + \beta \mu_e + \gamma$$

where $R_e$ is effective radius (in kpc), $\mu_e$ is the surface brightness at the effective radius (in magnitudes/arcsec²), and $\sigma$ is the central velocity dispersion (in km s⁻¹).

(a) In class, we wrote down the FP relation as: $R_e \propto \sigma^A \mu_e^B$. Using the conversion between $I_e$ (in luminosity/kpc²) and $\mu_e$ (in magnitudes/arcsec²), express the exponent, $\beta$, in terms of the exponent, $B$.

(b) The “tilt” of the FP can be expressed in terms of a relationship between mass and luminosity, of the form $L \propto M^\eta$, or $L = C M^\eta$. Re-express this equation in terms of the relationship between mass-to-light ratio, $M/L$, and mass, $M$.

(c) Given the relationship between mass and luminosity, and the FP equation, what do you derive for the relationship between the exponents, $\alpha$ and $\beta$? Next, express the exponent, $\eta$, in terms of $\alpha$ and $\beta$. [Note: the FP relation observed in the Coma cluster yields $\alpha = 1.25$, $\beta = 0.32$, and $\gamma = 8.97$].

(d) The FP is measured for elliptical galaxies at high redshift, and evolution is observed in its zeropoint, $\gamma$. This offset in zeropoint is commonly interpreted as evidence for evolution in the mass-to-light ratios of elliptical galaxies with lookback time. Express the evolution in the logarithmic mass-to-light ratio, $\Delta \log \frac{M}{L}$ in terms of the observed evolution in zeropoint, $\Delta \gamma$. If elliptical galaxies had lower M/L ratios in the past, would you expect them to be displaced above or below the $z \sim 0$ FP relation? Draw a diagram of the FP, with x-axis $\alpha \log \sigma + \beta \mu_e$ and y-axis $\log R_e$. On this diagram, show both the local FP and the direction in which an object moves as its M/L decreases.