Astro-286 - Week 2

1. Spherical collapse (30pt, 10pt each)

(a) In class I said that spherical collapse conserves angular momentum. Show that this is true.

(b) A typically observed radius of a collapsing and rotating cloud of a mass $1 \, M_\odot$ is $R_c \sim 0.1 \, pc$ with an angular velocity of $10^{-14} \, \text{rad sec}^{-1}$. What is the associated disk radius?

(c) Student Y missed the class and when trying to solve the above question he did the following: He derived the angular momentum of the cloud and found (like we did in class) that $J_c = \delta m R_c^2 \Omega$, he then said that since $\Omega^2 = GM/R_c^3$, so the specific angular momentum of the cloud is $j_c = J_c/\delta m = \sqrt{GM R_c}$. Similarly he found that the disk’s specific angular momentum is $j_D = J/\delta m = \sqrt{GM R_D}$. From conservation of angular momentum $j_c = j_D$ he found that $R_D = R_c$ and his answer to the above question was $0.1 \, pc$. What was his mistake?

2. Disks (70pt, 14pt each)

(a) Show that $\Sigma = \rho_0 \sqrt{2\pi h}$ (hint: remember what is the connection between $\Sigma$ and the vertical density distribution).

(b) In class we found that for a Keplerian motion

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left( \sqrt{R} \frac{\partial}{\partial R} \left[ \nu \Sigma \sqrt{R} \right] \right) ,$$

Find the expression for $v_R$.

(c) Steady state: In class we saw that the equation for the conservation of angular momentum is:

$$\frac{\partial (\Sigma R^3 \Omega)}{\partial t} = - \frac{\partial}{\partial R} \left( R v_R \Sigma R^2 \Omega \right) + \frac{1}{2\pi} \frac{\partial G_T}{\partial R} .$$

Assuming a steady state solution find the analytical expression the disk surface density, for a Keplerian disk, as a function of $R$ in terms of the mass accretion rate $\dot{M}$, the viscosity coefficient $\nu$ and the radius of the star $R_*$.  

Hint 1: Note that the accretion rate is defined as $\dot{M} = -2\pi R \Sigma v_R$.

Hint 2: Note that where $d\Omega/dR = 0$ the viscous stress vanishes. In a good approximation this can be set as a condition at the surface of the star.

(d) Disk surface temperature: The transport of energy, associated with viscous torque through in annulus is simply $G_T d\Omega/dR$, where $G_T$ is the torque we found in class. On one hand the dissipation rate per unit surface area of the disk, $D(R)$ is simply the viscous torque through in annulus over the two sided circumference (remember that the disk has two sides - what does it mean??). On the other hand for black body emission $D(R) = \sigma T_{disk}^4$. Assume a Keplerian disk, and use the result for $\Sigma$ you obtain to find the temperature profile of the disk as a function of $R$ in term of $\sigma, M_*, R_*, G$ and $\dot{M}$.
(e) **Disk central temperature**: The vertical energy flux $F(z)$ for an optically thick disk is given by the equation of radiative diffusion, i.e.,

$$F(z) = -\frac{16\sigma T^3}{3k\rho} \frac{dT}{dz}.$$  

(3)

Assume that all the energy dissipation happens at $z = 0$, and in that case $F(z) = \sigma T_{\text{disk}}^4$, which doesn’t depend on $z$. Find what is the temperature at the central of the disk $T_c$. Is the central temperature smaller or larger than $T_{\text{disk}}$?