Probable Detection of Cosmic Anisotropy by the
Cosmic Background Explorer (COBE)

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ABSTRACT

The Differential Microwave Radiometers (DMR) experiment on the COsmic Background Explorer (COBE) has seen a statistically significant anisotropy in the 2.73 K background radiation. The RMS amplitude of the sky, smoothed with a 10° beam, is $32 \pm 4 \mu K$ after the dipole is removed. The RMS quadrupole is $13 \pm 2 \mu K$ in the region with galactic latitude $|b| > 20°$. A scale-invariant or Harrison-Zeldovich spectrum is consistent with these values, and the best fitting amplitude for this spectrum gives an expected RMS quadrupole of $17 \pm 1.2$ (statistical) $\pm 5$ (systematic), where the dominant systematic uncertainty is galactic flux. This anisotropy seen by COBE is consistent with an unbiased cold dark matter model, with $H_0 = 50$, $\Omega_R = 0.1$, and $\Omega_{CDM} = 0.9$, which predicts an RMS quadrupole of $16 \mu K$. 
1 INTRODUCTION

The DMR instrument, described in detail by Smoot et al. (1990) measures anisotropy of the 2.73 K cosmic microwave background radiation (CMBR) at three different frequencies: 31.5, 53 and 90 GHz. Each frequency has two channels for redundancy, giving a total of six channels, denoted 31A, 31B, 53A, 53B, 90A and 90B. The orbit and pointing of COBE result in a complete survey of the sky every six months. Bennett et al. (1992) describe the calibration procedures, while preliminary results based on six months of data are given by Smoot et al. (1991).

Earlier experiments that have mapped almost the whole sky include the balloon surveys of Fixsen, Cheng & Wilkinson (1983), Lubin et al. (1985), and TBD, as well as the Relikt 1 experiment on the Prognoz 9 satellite (Klypin et al. 1987). These experiments have all seen the dipole anisotropy and galactic emission, but have only given upper limits on the quadrupole and smaller-scale anisotropies. The best pre-COBE limit on the RMS quadrupole was 82 μK given by Klypin et al., and under the assumption of a scale-invariant spectrum Klypin et al. derived a tighter limit of 56 μK.

In this paper we have constructed and analyzed three maps made from 1 year of data. The most sensitive map is one made from the average of the two 53 GHz channels, \((53A+53B)/2\), which we call the “sum” map. We have generated a “difference” map, \((53A-53B)/2\), to evaluate the noise in the sum map. Finally we have made a linear combination of all six channels that to first order cancels the signal from the galaxy. This “No Galaxy” (NG) map does show some isolated galactic signals generated by bright regions with anomalously flat or steep radio spectra, but the average galactic signal in the quadrupole is cancelled. These maps have a much lower noise level than the data given by an earlier experiment, and the upper limits previously obtained on \(l > 2\) anisotropies become very statistically significant detections.

2 ANALYSIS

The maps we use have been constructed using the sparse matrix analysis technique described by Torres et al. TBD. Systematic errors identified as major uncertainties in the preliminary results of Smoot et al. have been included in the fit and removed from the data. In particular, we have assumed that the magnetic effects on the radiometers can be modelled as a linear function of the Earth’s magnetic field at the spacecraft position. Signals from the Earth’s
limb have been estimated by coadding signals as a function of the azimuth and elevation of the Earth’s limb.

The “No Galaxy” maps have been constructed as a linear combination of the six channel maps by requiring that the average signal from the region with $\sin |b| < 0.1$ and $|l| > 30^\circ$ vanish; that the resulting map be in units of Planck brightness temperature, and that the noise be minimized. The requirements lead to a formula that basically forms the weighted sum of the 53 and 90 GHz channels, and then subtracts a fraction of the sum of the 31 GHz channels. Because of the subtraction, the NG map is about 1.7 times noisier than the sum map.

These maps have been subjected to several statistical analyses. The simplest involves smoothing the maps, and examining the decrease of the variance as the resolution decreases. True patterns on the sky are strongly correlated between adjacent pixels, while noise is uncorrelated. Thus if smoothing the map by averaging $4 \times 4$ pixel blocks reduces the noise by a factor of 4, then the map is dominated by noise. If the noise reduction factor is less than 4, that indicates the presence of real signals. A modification of this test uses a $7^\circ$ gaussian beam instead of square blocks of pixels. Monte Carlo tests have shown that noise is reduced to 0.27 of its original value in this smoothing, so a ratio of smoothed RMS to input RMS that is greater than 0.27 indicates a real signal. Figure 1 shows the sum map, smoothed with a $7^\circ$ gaussian beam, while Figure 2 shows the smoothed No Galaxy map.

A second analysis can only be done on the sum map. It compares the RMS of the sum map at a certain level of smoothing with the RMS of the difference map at the same smoothing level. This method has an easily determined uncertainty. The RMS of the true sky signal is given by

$$RMS(sky)^2 = RMS(Sum)^2 - RMS(Diff)^2$$

and the uncertainty in the RMS of the sky is given by

$$var(RMS(sky)^2) = \sqrt{(2/N)(RMS(Sum)^4 + RMS(Diff)^4)}$$

where N is the number of degrees of freedom in the map. Since we have always removed a monopole+dipole fit, N is 4 less than the number of pixels or pixel blocks in the analyzed area. In the case of the gaussian smoothed maps, the number of degrees of freedom is taken to be 4 less than 0.27$^2$ times the number of input pixels in the analyzed area. Table 1 gives the variance of the sum and difference maps at various smoothing levels, and the derived values of the RMS of the sky, with 1σ errors. The 53 GHz values in this table are Rayleigh-Jeans brightness temperatures, which are 7% smaller than Planck brightness temperature differences.
A third analysis calculates the correlation function of the sky by finding the cross-correlation between different channels. We have three sensitive channels, the 53A, 53B and the 90B. Figure 3 shows the three cross-correlations formed from these three channels as a function of angular separation. Only the region with \( |b| > 30^\circ \) has been used. All data have been converted into Planck brightness temperatures. The correlation function from a scale invariant spectrum with an expected RMS quadrupole of 17 \( \mu \text{K} \) is shown as the solid curve. Because of the asymmetry introduced by the galactic plane cut, this curve has been evaluated using a Monte Carlo program. Ten different sets of 100 random skyls have been used to find ten different correlation functions, which are all plotted as curves in the Figure.

A fourth statistical test is the calculation of the RMS quadrupole of the maps. The quadrupole components in galactic coordinates and \( Q_{RMS} \) for quadrupole fits to the sum map and the No Galaxy map are given in Table 2. Note that the sum map needs to be multiplied by 1.07 to convert Rayleigh-Jeans temperature differences into Planck temperature differences: the Table gives Rayleigh-Jeans values. The smallest \( Q_{RMS} \) is 13\( \pm 2 \) \( \mu \text{K} \) in Planck temperature units, where the error is purely statistical.

The most sensitive statistical test for anisotropy uses an assumed form of the correlation function \( C(\theta) \), with an adjustable amplitude. We have used the correlation function corresponding to a scale-invariant or Harrison-Zeldovich spectrum, where the amplitudes associated with the spherical harmonics \( Y_{\ell m} \) are independent gaussian random variables with variances \( \propto [\ell(\ell+1)]^{-1} \). Since we cut the galactic plane out of our maps before analyzing our data, the correlation function depends on both positions being correlated, instead of just their separation. We have evaluated the appropriate correlation function by generating more than 20,000 random sky patterns, removing the galactic plane, fitting and subtracting a dipole and monopole from the high galactic latitude data, and then averaging the cross-products. Since using the full resolution data would give millions of cross-products, we have averaged together 4 \( \times \) 4 blocks of pixels before correlating. This gives a total of 384 pixel blocks on the sky, and we have deleted the 128 of these closest to the galactic plane, giving a final 256 \( \times \) 256 symmetric correlation matrix \( C_{ij} \). We have normalized the simulated skymaps used to generate \( C_{ij} \) to have an RMS quadrupole of unity. The expected value of the product of the two temperatures is

\[
\langle T_i T_j \rangle = A^2 C_{ij} \pm \sigma_i \sigma_j
\]  

where the uncertainty assumes that the correlation is small. A weighted least squares fit to evaluate the adjustable amplitude \( A^2 \) gives

\[
A^2 = \sum_{i<j} W_{ij} C_{ij} T_i T_j / \sum_{i<j} W_{ij} C_{ij}^2
\]
where the weight is given by
\[ W_{ij} = 1/(\sigma_i^2 \sigma_j^2) \] (5)

The uncertainty in the amplitude is given by
\[ \text{var}(A^2) = (\sum_{i<j} W_{ij} C_{ij}^2)^{-1} \] (6)

This analysis gives the following results: The sum map has an amplitude of \( A^2 = 201 \pm 19 \mu K^2 \), while the No Galaxy map has an amplitude of \( A^2 = 288 \pm 39 \mu K^2 \). Since the sum map is in Rayleigh-Jeans units, its \( A^2 \) needs to be multiplied by 1.15 before comparing these two results. Thus the two maps differ by only 1.25\( \sigma \) in their estimated amplitudes. We have adopted the result from the No Galaxy map in the following discussion, even though it is noisier, because the galaxy subtraction should reduce the effect of the most worrisome systematic error, which is galactic emission. Even so, the resultant value for the expected value of \( Q_{\text{RMS}} = 17 \pm 1.2 \mu K \), which is remarkably precise. There is apparently a residual effect of the galaxy on the quadrupole fits, even for the No Galaxy map, so we have assigned a possible systematic error of \( \pm 5 \mu K \) to this result. Note that even though the statistical error is very small, there can be a large uncertainty in relating the observed value \( A^2 \) or \( Q_{\text{RMS}} \) to the primordial potential fluctuations in the Universe, due to the small number of degrees of freedom in the single sky that we can observe. In the case of the quadrupole, for which \( Q^2 \) follows the \( \chi^2 \) distribution with 5 degrees of freedom, this uncertainty is \( \pm 30\% \). A Monte Carlo calculation shows that the scatter in the value of \( A \) derived from simulated maps made with a known value of \( A = 16 \mu K \) is \( \pm 4 \mu K \) 1\( \sigma \), or \( \pm 25\% \). Thus there is no incompatibility between \( Q_{\text{RMS}} = 13 \mu K \) and \( A = 17 \mu K \). The COBE results presented here are consistent with a Harrison-Zeldovich spectrum.

Note that for a Harrison-Zeldovich spectrum, the expected ratio between the RMS observed with a 10° gaussian beam and \( Q_{\text{RMS}} \) is 2. Thus the RMS determined from the smoothing and sum-difference techniques is also consistent with \( Q_{\text{RMS}} \) and a Harrison-Zeldovich spectrum.

3 DISCUSSION

The anisotropies we have detected are all at scales that are very large compared to the scales of the inhomogeneities studied by angular correlation and redshift surveys of galaxies. The smoothed maps have an effective beam size of 10°, which corresponds to 100,000 km/sec for \( \Omega = 1 \). To compare the observed anisotropy with galaxy surveys, we assume a primordial spectrum with the Harrison-Zeldovich form, as predicted by the inflationary
scenario. Holtzman (1989) gives predicted quadrupole amplitudes for about 100 different models with baryons plus hot or cold dark matter. For models with baryon densities fixed by standard hot Big Bang nucleosynthesis, and the rest of the closure density supplied by cold dark matter, the predicted quadrupole can be approximated by

\[ Q = \frac{800 \, \mu K}{b_8 \times H_0} \]  

(7)

where \( b_8 \) is the bias factor in spheres of radius 800 km/sec. Thus unbiased CDM with \( H_0 = 50 \) is consistent with the observed anisotropy.

The APM galaxy survey by Maddox et al. (1990) and the QDOT IRAS redshift survey by Efstathiou et al. (1990) have suggested the existence of a perturbation spectrum with more power on large scales (10,000 km/sec) than that provided by a Harrison-Zeldovich spectrum. Clearly the COBE DMR results do not require this excess power. However, the observed anisotropy is equal to the largest anisotropy predicted by reasonable CDM models, so a spectrum with a modestly enhanced power at large scales, combined with a \( b_8 \times H_0 \) somewhat larger than 50, could also fit the DMR data. For example, models with \( b_8 = 1 \), \( H_0 = 100 \) and a cosmological constant large enough to give a reasonable age of the Universe also predict the observed anisotropy. Holtzman (1989) found an expected \( Q_{RMS} = 17 \, \mu K \) for a model with \( H_0 = 100 \), \( \Omega_{\text{vac}} = 0.8 \), \( \Omega_{\text{CDM}} = 0.18 \), and \( \Omega_B = 0.02 \).


Table 1: Map RMS for $|b| > 19.5^\circ$ in $\mu$K

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>Smoothing</th>
<th>RMS(sum)</th>
<th>RMS(diff)</th>
<th>RMS(sky) sum-diff</th>
<th>RMS(sky) smoothing</th>
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<tr>
<td>53</td>
<td>1x1</td>
<td>146</td>
<td>141</td>
<td>$39^{+7}_{-10}$</td>
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<td>53</td>
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<td>71</td>
<td>$32^{+5}_{-6}$</td>
<td>$29^{+5}_{-6}$</td>
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<td>53</td>
<td>4x4</td>
<td>46</td>
<td>34</td>
<td>$31^{+4}_{-4}$</td>
<td>$29^{+3}_{-4}$</td>
</tr>
<tr>
<td>53</td>
<td>7$^\circ$</td>
<td>50</td>
<td>39</td>
<td>$32^{+4}_{-4}$</td>
<td>$31^{+3}_{-3}$</td>
</tr>
<tr>
<td>NG</td>
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<td>242</td>
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<td></td>
<td></td>
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<tr>
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<td>$26^{+7}_{-9}$</td>
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<tr>
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<td>7$^\circ$</td>
<td>72</td>
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<td>$27^{+7}_{-10}$</td>
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Table 2: Quadrupole Fits in $\mu$K

<table>
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<tr>
<th>$\nu$</th>
<th>$b_{cut}(^\circ)$</th>
<th>$Q_{RMS}$</th>
<th>$Q_1$</th>
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<td>15</td>
<td>16</td>
<td>5</td>
<td>17</td>
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Figure 1: Smoothed, dipole removed average of 53A and 53B

Figure 2: Smoothed, dipole removed No Galaxy map
Figure 3: Cross-correlations for $|b| > 30^\circ$ between the 53A & 53B (filled squares), 53A & 90B (filled circles), and the 53B & 90B (open squares), plus Monte Carlo predictions for a scale invariant spectrum with an expected quadrupole amplitude of 17 $\mu$K