FINE STRUCTURE OF SOLAR ACOUSTIC OSCILLATIONS DUE TO ROTATION

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Ledoux (1951) predicted that rotation causes a fine structure in the spectrum of stellar oscillations. Assuming rigid rotation, he attributed the resultant splitting to linear advection which lifts the degeneracy of each individual oscillation multiplet yielding a spectrum of uniformly spaced fine structure centered about each unperturbed frequency. A more complicated rotation law implies a more complicated spectrum. Thus, the observation of the fine structure of stellar oscillations can be used, in principle, to determine the rotation law inside the star. Our purpose here is to predict the nature of the fine structure of high order, low degree five minute period solar oscillations following from various postulated forms of spherical rotation. We include the first and second order effects of rotation.

The frequency splitting caused by linear advection, assuming spherical rotation, is given by (Hansen, Cox and Van Horn, 1977)

$$\omega_1^{n\ell m} = \frac{m\int_0^{R_\odot}\Omega(r)\left\{-\left(y_{n\ell}^2(r)+\ell(\ell+1)z_{n\ell}^2(r)\right)+2y_{n\ell}(r)z_{n\ell}(r)+z_{n\ell}^2(r)\right\}\rho r^4 dr}{\int_0^{R_\odot}\left(y_{n\ell}^2(r)+\ell(\ell+1)z_{n\ell}^2(r)\right)\rho r^4 dr} \quad (1)$$

where $\Omega(r)$ is the rotation rate and the normal coordinate is given by

$$\vec{\xi}_{n\ell m} = \left[\vec{r}\; y_{n\ell}(r)Y_\ell^m(\theta,\phi)+r\; z_{n\ell}(r)\vec{\nabla}_H Y_\ell^m(\theta,\phi)\right]e^{i\omega_o^{n\ell}t} \quad (2)$$

The quantity $\omega_o^{n\ell}$ is the angular frequency of the oscillation in the absence of rotation. The remaining terms in Equations (1) and (2) have their usual meanings. The expression for the frequency splitting, $\omega_2^{n\ell m}$, due to the second order effect of rotation is lengthy. Therefore, we present the asymptotic part of the expression which is valid for acoustic modes (high order and/or high degree),

$$\omega_2^{n\ell m} \simeq K_{n\ell}^m\omega_o^{n\ell}\int_0^{R_\odot}\left\{\frac{d}{dr}(\varepsilon r)y_{n\ell}^2+\ell(\ell+1)z_{n\ell}^2\varepsilon-\frac{1}{3}\frac{\Omega^2 r}{g}\mu\right.$$
$$\left.\left[\left(y_{n\ell}^2+\ell(\ell+1)\right)z_{n\ell}^2+\left(2\mu+\frac{d\ell n\mu}{d\ell n r}\right)y_{n\ell}\left(z_{n\ell}-y_{n\ell}\frac{g}{\omega_o^2 r}\right)\right]\right\}\rho r^4 dr\Big/$$
$$\left[\int_0^{R_\odot}\left(y_{n\ell}^2+\ell(\ell+1)z_{n\ell}^2\right)\rho r^4 dr\right] \quad , \quad (3)$$

where g is the local gravity and

$$\mu = \frac{d\ln\Omega(r)}{d\ln r} \tag{4}$$

and

$$K_{n\ell}^{m} = \frac{\ell(\ell+1)-3m^2}{4\ell(\ell+1)-3} \tag{5}$$

where $K_{n\ell}^{m}$ represents the result of the angular integration. The quantity ε is given by

$$\varepsilon = \frac{\Omega^2 r}{3g} + \frac{\phi_2}{gr} \tag{6}$$

where ϕ_2 is the perturbation of the gravitational potential due to rotation. We wish to thank Douglas Gough (1983) for providing us his equations which enabled us to find our error in the oral presentation. Thus, the results shown here are different from those presented at the conference.

For the purpose of calculation, we assume that the internal solar rotation rate is twice the surface rate from the center to R_1 and smoothly, monotonically decreases to the surface value in going from R_1 to R_2. We further assume that the rotation rate equals the surface rate from R_2 to the surface (see Figure 1).

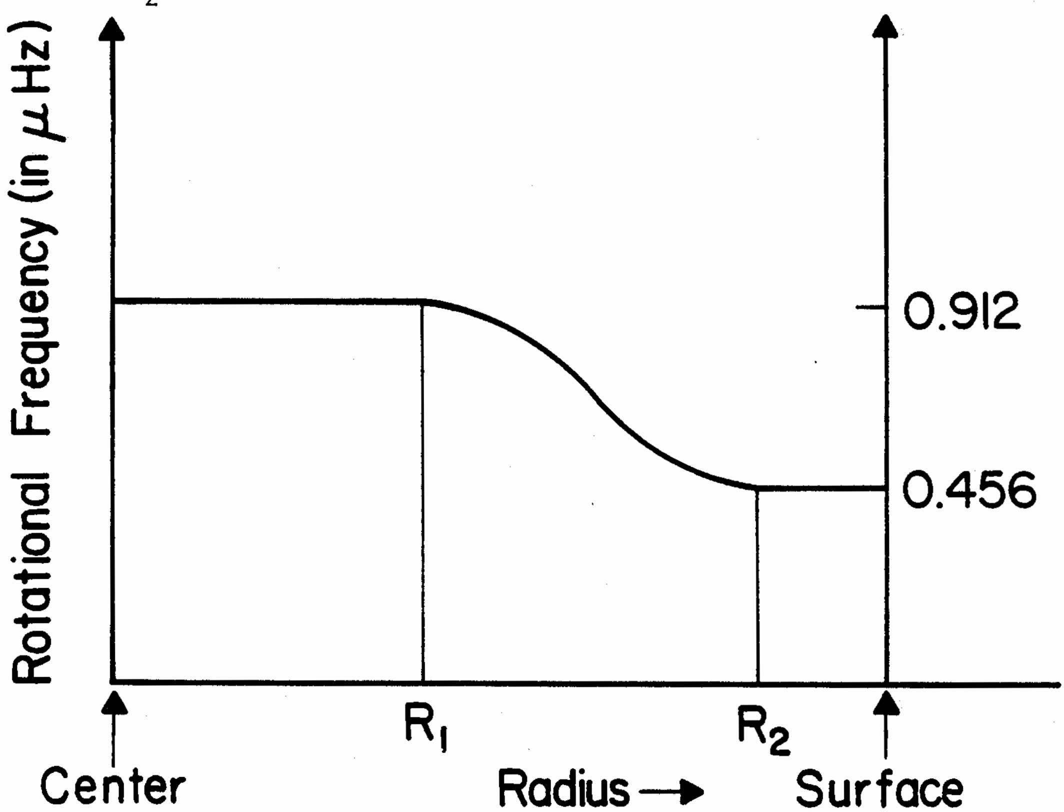

Figure 1. The Internal Rotational Frequency as a Function of Radius.

The predicted splittings which follow from the implementation of the assumed rotation law are given for the P_{22} $\ell = 1$ five minute period mode for various values of R_1 and R_2 (see Table 1).

Table 1. Predicted First and Second Order Rotational Splitting (in μHz) between the m = 0 and 1 components of the P_{22} $\ell = 1$ mode.

R_1/R_o	R_2/R_o	$\frac{\omega_1}{2\pi}$	$\frac{\omega_2}{2\pi}$
0.998	0.999	-0.89	1.30
0.990	0.999	-0.89	0.15
0.720	0.999	-0.72	-0.03
0.719	0.720	-0.64	-0.03
rigid rotation		-0.45	-0.03

The second order effect of rotation is large only if there is a sharp gradient in Ω very near the surface. The large effect is a result of the sharp gradient in the region where the splitting kernel is largest. The splitting kernel is the integrand in Equation (1) if $\Omega(r) = 1$. For the other cases, including a sharp gradient at the base of the convection zone the second order effect of rotation is small compared to the first order effect of rotation.

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References:

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