Astro Lab: Rotation Curves

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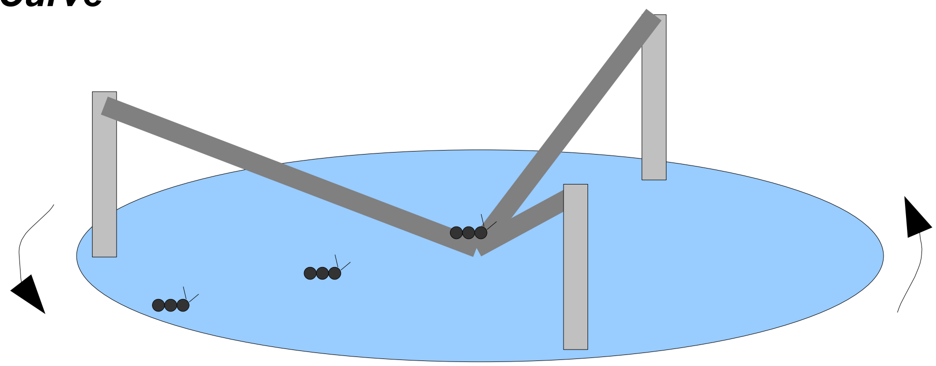
At the end of this, you should be able to:

- Explain the concept of a rotation curve
- Describe the utility of rotation curves in astronomy
- Calculate a rotation curve, given a mass distribution
- Make inferences about a mass distribution given a rotation curve

A rotation curve is just a graph that tells you how the speed of things in a spinning system relates to the distance of those things from the rotation axis. When combined with Newton's and Kepler's laws of gravitation and orbital dynamics, a rotation curve is a powerful tool that can be used to measure the distribution of matter in a rotating system such as a solar system or galaxy. To introduce the topic, however, we'll start in a more familiar environment: the playground.

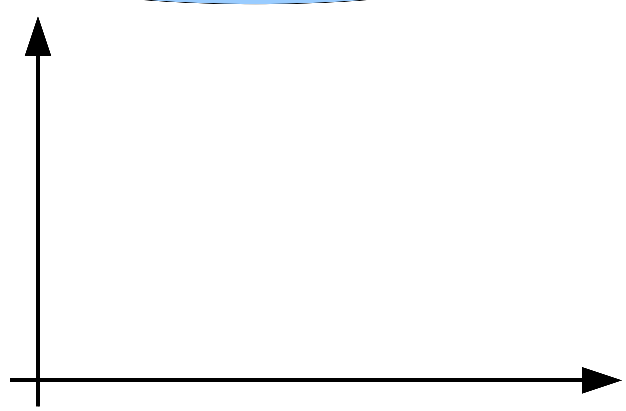
Merry-go-round Rotation Curve



Consider the merry-go-round drawn to the right. Since it's the middle of the day and all the children are in school, you can see that the ants have free rein in the playground – and there's nothing they like better than a ride on the merry-go-round (never mind how they get it spinning).

At the moment, three ants are riding the spinning merry-go-round. It's been well spun-up, and the outer edge of the merry-go-round is moving at 5 m/s.

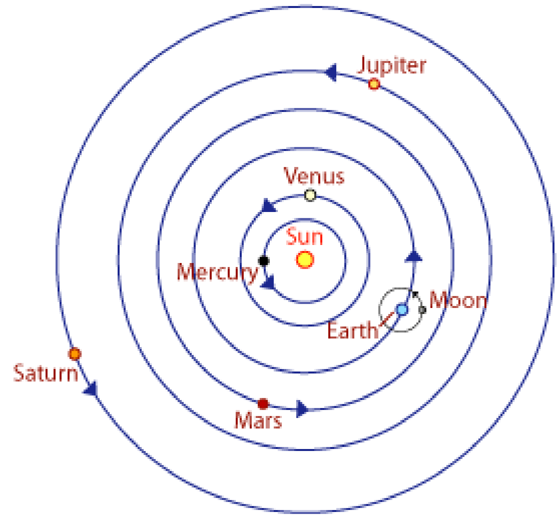
How fast are each of the three ants travelling?



Plot the position and speed of each of the three ants on the axes at right, and connect them with a "best-fit" curve. What does this tell you about rotation speeds on a merry-go-round?

This is an example of what we commonly call "**solid body rotation**." It occurs when all the material in a solid spinning object – a merry-go-round, a bicycle wheel, or the Earth itself – goes around together.

Solar System Rotation Curve



Now let's travel a bit farther afield and consider the movement of objects in our solar system around the sun. Note that the schematic drawing to the right is *not* to scale! First, let's start with the nearest and dearest of all these objects: the Earth.

(a) How far away from the Sun is the Earth?

(b) How far does the Earth travel in one orbit around the Sun?

(c) How long does it take to go around the sun once?

(d) Given these quantities, how fast does the Earth travel around the Sun?

You should find that the Earth is moving around the Sun at a fairly brisk pace. Why don't you feel it?

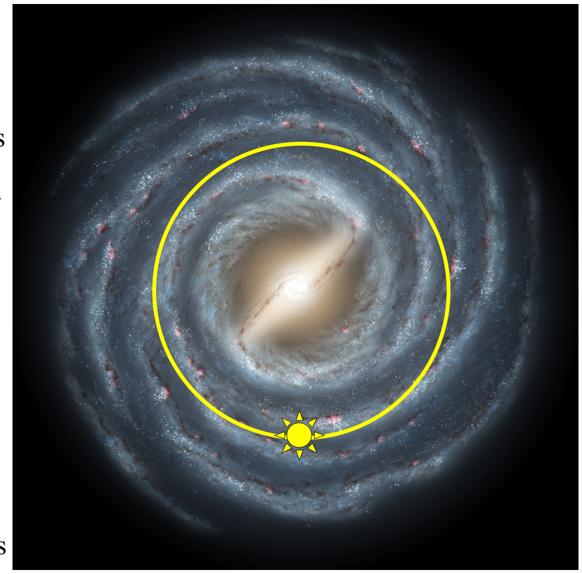
Pick three other planets in the solar system; use your textbook, the internet, or your instructor to find the orbital radii and orbital period ("planetary year") for each of these planets. Then follow the same steps you did for the Earth to find how fast each of those planets goes as it orbits the sun.

Once you have determined the planets' speeds, plot their distance from the sun and their speed on the axes provided at right and connect them with a "best-fit" curve. How does this curve differ from the solid-body rotation curve of the merry-go-round?

The force acting on a planet of mass "m" is due to the gravity of everything lying within its orbit – in the solar system, the Sun is by far the heaviest object. Since the planets' orbits are nearly circular, they undergo uniform circular motion and experience a centripetal force. These two forces must be equal; therefore, $\frac{G\,m\,M_{sun}}{r^2}=\frac{m\,v^2}{r}$. Using the values you calculated above for one of the planets (*not* Earth), calculate the mass of the sun from the planet's velocity and orbital radius.

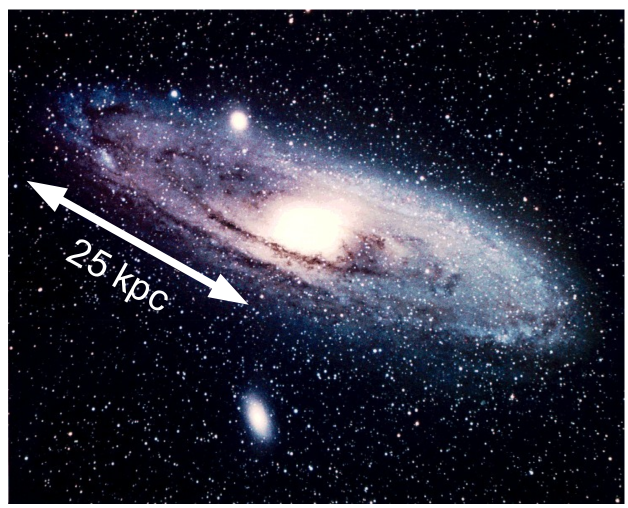
In this way, we can use the speed of an orbiting object to measure the amount of mass enclosed by its orbit.

Rotation Curves of Galaxies



Using spectroscopy, astronomers can measure the Doppler shift of light from stars in nearby galaxies. In addition, they can use the Doppler shifts of radio waves from neutral hydrogen to measure the speed of rotation in our own Galaxy, the Milky Way. This then poses the reverse of the previous problems – instead of calculating a rotation curve from a given mass, astronomers want to calculate how much mass is in a galaxy once they have measured the speed of stars orbiting within it. Starting again from the equation of circular gravitational motion -- $\frac{G\,m\,M_{enclosed}}{r^2} = \frac{m\,v^2}{r}$ -- rewrite it to show how you would calculate the enclosed mass of a given system.

The sun moves at 220 km/s around the center of our Galaxy, which is 8500 pc away. How much material in the Milky Way lies within the Sun's orbit (drawn above)?



The Andromeda Galaxy, M31, is the nearest large galaxy to the Milky Way. It is also somewhat larger than the Milky Way; the disk of stars in it extends out roughly 25 kpc from the center, as shown to the right (the Milky Way is only 15 kpc in radius).

The recently-measured rotation curve of M31 is shown to the lower right (the set of points indicated by the bold, black diamonds). Use this graph to determine the amount of mass contained within the orbit of a star near the edge of the Andromeda Galaxy (i.e., about 25 kpc from the center).

From the picture of M31, how much more mass would you expect to be enclosed by an orbit 35 kpc in radius? Why?

Use the rotation curve at right to calculate the mass enclosed by an orbit of radius 35 kpc. How does this compare with your estimate above? Reflect on this result.