

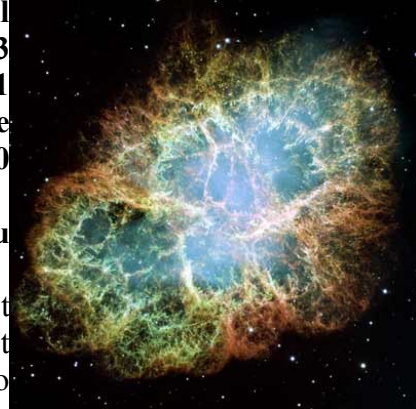
Astronomy 82 - Problem Set #3

Due: Friday, April 25, 2008 at the beginning of class.

Reading: Finish chapter on variable stars. Read chapter on compact stars.

Problems:

- 1) In 1983, the radius of the Crab Nebula (a well known supernova remnant) was about 3 arcminutes. It is expanding at a rate of 0.21 arcseconds per year. The radial velocity of the expanding gas compared to the central star is 1300 km/sec.



- a) What is the distance to the nebular if you assume that the expansion is symmetric.

The angular size of an astronomical object is just given by $\theta = S/d$ where S is the size of the object and d is the distance to it. The easiest way to do this problem is to recognize that this relation is unchanged if you are instead talking about *changes* in S and theta. Namely, $\Delta\theta = \Delta S/d$, which can be rearranged to give $d = \Delta S / \Delta\theta$.

How quickly are the physical size and the angular size changing? In one year, the diameter S of the nebula increases by $2 \times 1.3E+6 \text{ m/s} \times 3E+7 \text{ s/yr} = 8.2E+13 \text{ m}$. The angular size changes in one year by $2 \times 0.21'' = 2.0E-6 \text{ rad}$. The distance is then:

$$\Delta\theta = \frac{\Delta S}{d} = \frac{8.2 \times 10^{13} \text{ m}}{2.0 \times 10^{-6} \text{ rad}} = 4.0 \times 10^{19} \text{ m} = 1300 \text{ pc}$$

Of course, the actual expansion is highly nonuniform (as the picture above demonstrates), so this is only an approximate answer. Any reliable resource will tell you that the Crab Nebula is about 2000 pc away. For such a simple estimate, we've done well to get within a factor of two.

- b) How long ago did the supernova occur?

There are actually several ways to answer this question. Are we asking how long ago the supernova was first *seen*, or how long ago it exploded in absolute terms?

To find when it was first seen, consider the following: if there are 10 gallons of water in a bathtub and water has been poured in at 1 gallon per minute, how long has the water been on? The answer to part (b) here involves the same basic concept in almost any "rates" problem:

"quantity" = "rate" x "time"

In this case, the **quantity** we are given is an angular radius of 3 arcminutes and the **rate** is the increase in angular size: 0.21'' per year. Set it up the same as the bathtub problem:

$$t = \frac{\text{quantity}}{\text{rate}} = \frac{\theta}{\omega} = \frac{3 \text{ arcmin}}{0.21 \text{ arcsec/yr}} \times \frac{60 \text{ arcsec}}{1 \text{ arcmin}} = 857 \text{ yr} \approx 900 \text{ yr ago.}$$

(Note that we only have one significant figure in this problem!) So we see that the supernova had been expanding for about 900 years in 1983 – therefore the supernova must have been seen to explode around the year $(1983 - 900) = 1083 \sim 1100$ AD. Again, this number is no secret – the Crab Nebula was observed by astronomers in several countries when its light reached earth in 1054 AD.

So, how long before the supernova was seen to explode did it actually go off? We'll use our calculated distance of 1300 pc, which corresponds to 4200 light years. Since light travels at the speed of light (!), it must have taken the supernova's light roughly 4200 years to reach earth. Combining that with our calculation of when the supernova was first observed, you can see that the supernova must have occurred $(900 \text{ yr} + 4200 \text{ yr}) = 5100 \text{ yrs} \sim 5000$ years ago – around the time of Earth's Early Bronze Age.

c) If the absolute magnitude of the supernova was -18 mag, what was its apparent magnitude from the Earth?

Any astronomer worth their salt should realize that this is a straightforward application of the magnitude-distance relation:

$$m - M = 5 \log(d/10 \text{ pc})$$

Which we can rearrange to give the apparent magnitude:

$$m = M + 5 \log(d/10 \text{ pc})$$

$$m = -18 + 5 \log(1300 \text{ pc}/10 \text{ pc}) = -18 + 5 \log(130) = -7.4$$

Easily bright enough to have been seen in the daytime!

2) If a type Ia supernova is observed to have a peak magnitude of 20 mag, then what is its luminosity distance in parsecs? Assume that a typical type Ia supernova has a peak absolute magnitude of -19.6 mag.

Let's apply that same, wonderful, magnitude-distance relation:

$$m - M = 5 \log(d/10 \text{ pc})$$

except that this time we'll rearrange it to solve for the distance modulus:

$$d = (10 \text{ pc}) \times 10^{\frac{m-M}{5}} = (10 \text{ pc}) \times 10^{\frac{20+19.6}{5}} = 8.3 \times 10^8 \text{ pc} = 830 \text{ Mpc}$$

(Note that as we go out to increasingly large “cosmological” distances, these simple equations no longer apply exactly; we'll talk about this more later in the course. See http://www.astro.ucla.edu/~wright/cosmo_02.htm)

(b) Approximately how long ago did the supernova occur?

The simplest way to do this is just divide the distance by the speed at which the light has been traveling:

$$t = \frac{d}{c} = \frac{830 \text{ Mpc}}{c} = \frac{8.3 \times 10^8 \text{ pc}}{c} \times \frac{3.26 \text{ ly}}{1 \text{ pc}} = 2.7 \times 10^9 \text{ ly}/c = 2.7 \text{ Gyr}$$

- 3) **A white dwarf is the remnant of a dead star which is supported by electron degeneracy pressure instead of nuclear fusion. The maximum mass that can be supported by this pressure is $1.44 M_{\text{solar}}$. (For each calculation, you can assume the star has uniform density.)**

- a) If such a maximum mass white dwarf has a radius of 5400 km (slightly smaller than the Earth), what is its gravitational binding energy (potential energy)?**

If we assume that the white dwarf has constant density (not actually true, but this is an introductory course) then we can use the relation for the potential energy of a constant-density sphere:

$$U = -\frac{3}{5} \frac{G M^2}{R} = -\frac{3}{5} \times \frac{(2/3 \times 10^{-10}) \times (1.44 \times 2 \times 10^{30} \text{ kg})^2}{5.4 \times 10^6 \text{ m}} = -6.1 \times 10^{43} \text{ J}$$

Notice that the energy is negative – if it were positive, the star would be unbound and fly apart.

- b) If such an object gains a little more mass, then it will collapse until a neutron degeneracy pressure dominates. What is the potential energy if the same $1.44 M_{\text{solar}}$ object collapses to the size of a neutron star ($R \sim 8 \text{ km}$)?**

Just use the same equation as you did above. Since $U \propto 1/R$, we have the result that

$$U_{\text{NS}} = U_{\text{WD}} \frac{R_{\text{WD}}}{R_{\text{NS}}} = (-6.1 \times 10^{43} \text{ J}) \frac{5400 \text{ km}}{8 \text{ km}} = -4.1 \times 10^{46} \text{ J}$$

- c) How much energy is released in the collapse?**

The energy released is

$$\Delta E = E_{\text{initial}} - E_{\text{final}} = 4.1 \times 10^{46} \text{ J}$$

Most of this is actually released in the form of neutrinos, but a substantial portion is released as visible light – this is what we see, and how we detect supernovae and other bright outbursts!