A82 Sample problems:

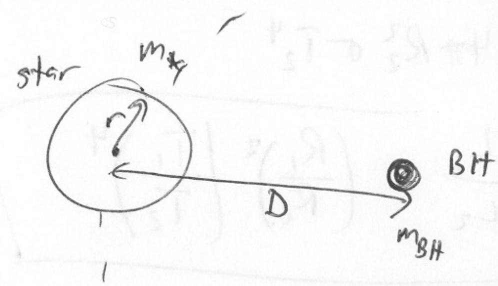
1) A star of radius r orbits a supermassive black hole of radius R at a distance D from the center, orbiting in a time P.

a) Find the mass of the black hole.

b) When a star is "tidally disrupted," the star is torn apart because the gravity of a more massive object starts to exceed the force of gravity holding the star together. How massive must the star be to ensure that it is not tidally disrupted?

a) Many people tried to memorize Kepler's Law, but you can just use centripetal accel:

$$\frac{v^2}{D} = \frac{G M_{BH}}{D^2} \Rightarrow \boxed{M_{BH} = \frac{v^2 D}{G}}$$



b) The star's gravity must be stronger than the BH's at the surface!

$g_* \geq a_{BH}$ (assume $D \gg r_*$)

$$\frac{G m_*}{r_*^2} \geq \frac{G M_{BH}}{D^2} \Rightarrow \boxed{m_* \geq \left(\frac{r_*}{D}\right)^2 M_{BH}}$$

2) A G-type star rotate with an angular velocity w. Estimate how fast will it rotate as a white dwarf after losing roughly half its mass (and its radius decreases by a factor of 100).

Here we assume angular momentum is conserved and that the stars' moments of inertia are each $I = \frac{2}{5} M R^2$

$L_1 = L_2 \qquad L = I\omega$

$$I_1 \omega_1 = I_2 \omega_2 \Rightarrow \frac{2}{5} M_1 R_1^2 \omega_1 = \frac{2}{5} M_2 R_2^2 \omega_2 \Rightarrow \boxed{\omega_2 = \frac{M_1}{M_2}\left(\frac{R_1}{R_2}\right)^2 \omega_1 = 20{,}000\, \omega_1}$$

3) A supernova recently observed at apparent magnitude m was reported to be R times brighter than Earth's sun and a distance D away. Estimate the amount of extinction between the supernova and earth.

~~Thus~~ Without extinction, $m = M + 5 \log(D/10\text{pc})$

Extinction blocks light, so apparent magnitude is greater:

$$m_x = M + 5 \log(D/10\text{pc}) + A$$

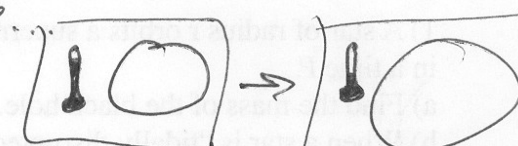
$$M - M_\odot = -2.5 \log\left(\frac{R L_\odot}{L_\odot}\right) \Rightarrow M = M_\odot - 2.5 \log(R)$$

Thus, $\boxed{A = m_x - (M_\odot - 2.5 \log(R)) - 5 \log(D/10\text{pc})}$

4) A Cepheid variable star increases in radius from R1 to R2 while its surface temperature decreases from T1 to T2. How much does its luminosity change?

Black body radiation is the key here.

generally, $L = 4\pi R^2 \sigma T^4$

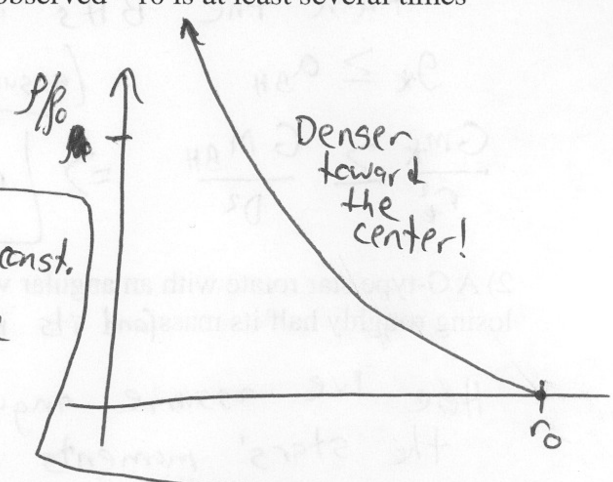


$L_1 = 4\pi R_1^2 \sigma T_1^4$

$L_2 = 4\pi R_2^2 \sigma T_2^4$

$$\boxed{\frac{L_1}{L_2} = \left(\frac{R_1}{R_2}\right)^2 \left(\frac{T_1}{T_2}\right)^4}$$

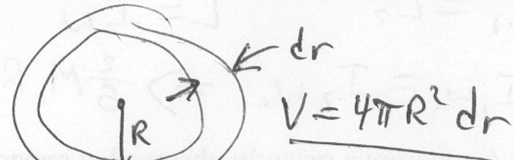
5) The mass of a galaxy is dominated by its dark matter halo. Assume a dark matter halo has a density distribution of p(r) = p0(r0/r – 1) (this is, very roughly, what is observed – r0 is at least several times larger than the visible diameter of a galaxy).
a) Sketch this density distribution.
b) How much dark mass is enclosed within a volume of radius r?
c) Calculate the rotation curve implied by this mass distribution.



b). Not $\cancel{\frac{4\pi}{3} r^3 \rho}$!! Density isn't const.

Instead, you have to add up the mass of each spherical shell:

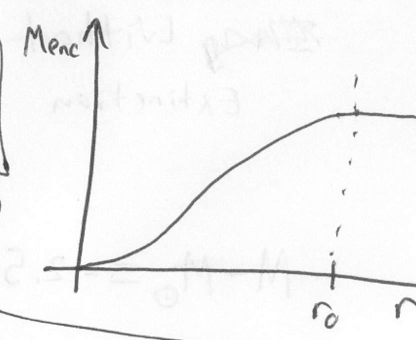
$$dM_{shell} = 4\pi R_{shell}^2 \, \rho(R_{shell})\, dr$$



$$M(r) = \int_0^r 4\pi r^2 \rho_0 \left(\frac{r_0}{r} - 1\right) dr$$

$$= 4\pi\rho_0 \int_0^r \left(r_0 r - r^2\right) dr$$

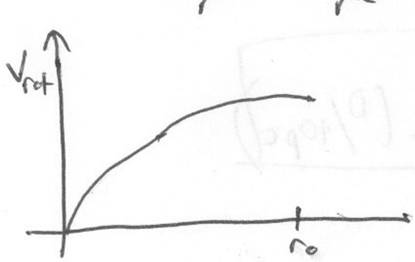
$$= 4\pi\rho_0 \left(\frac{r_0 r^2}{2} - \frac{r^3}{3}\right)\Big|_0^r = \boxed{4\pi\rho_0\left(\frac{r_0 r^2}{2} - \frac{r^3}{3}\right)}$$



c). As always, assume circular motion...

$$\frac{v^2}{r} = \frac{G M_{enc}}{r^2} \Rightarrow v = \sqrt{\frac{G M_{enc}}{r}}$$

$$\boxed{v = \sqrt{4\pi G\rho_0}\sqrt{\frac{r_0 r}{2} - \frac{r^2}{3}}}$$



← sort of like what we see...