Astronomy 82 - Problem Set #5

Due: Friday, June 6, 2008 in class or to Ian by 12 pm

Problems:

1) **If the rotation rate of the Galaxy remains at about 220 km/sec out to a radius of at least 15 kpc, then what is the minimum mass of the Galaxy?**

The galaxy is rotating circularly... so we can just equate gravitational and centripetal accelerations to solve for the total galactic mass:

$$a_g = a_c$$
$$\frac{G M_{gal}}{r^2} = \frac{v^2}{r}$$
$$M_{gal} = \frac{v^2 r}{G}$$
$$M_{gal} = \frac{(2.20\times10^5\,\text{m/s})^2 \times (15\,\text{kpc}\times3.09\times10^{19}\,\text{m/kpc})}{6.67\times10^{-11}}$$
$$M_{gal} = 3.36\times10^{41}\,\text{kg}\times\frac{M_{sun}}{2.0\times10^{30}\,\text{kg}} = 1.7\times10^{11}\,M_{sun}$$

2) **The Virgo cluster of galaxies is at a distance of 65 million light years, and has a redshift equivalent to 1400 km/sec. Assuming the speed has remained constant throughout the past and the cluster has no tangential motion, then when were we and the cluster at the same location?**

An analogy: consider a car driving away from you at 60 miles per hour. If the car is 120 miles away, how long has it been driving away? To solve this, you just use the old "distance = rate x time" equation... that's really all the Hubble Law is!

So we solve:

$$d = r\,t$$
$$t = \frac{d}{r}$$
$$t = \frac{65\,\text{Mly}}{1400\,\text{km/s}}$$
$$t = \frac{6.5\times10^7\,\text{ly}\times(9.5\times10^{15}\,\text{m/ly})}{1.400\times10^6\,\text{m/s}} = 4.41\times10^{17}\,\text{s}$$
$$t = 4.41\times10^{17}\,\text{s}\times3.16\times10^7\,\text{s/yr} = 14\,\text{billion years}$$

3) Our Galaxy ($7x10^{11}$ M_{sun}) and the Andromeda galaxy (M31, $m=10^{12}M_{sun}$) are by far the most massive members of the Local Group. If these are separated by 690 kpc and are in circular orbits about the common center of mass (which they're not), what would be our distance to that center of mass? What would our orbital period be (use the equations for binary stars)?

To find the distance to the center of mass, we'll first say that we (the Milky Way) is at r=0, and that M31 is at $r=r_{31}$. Then the location of the center of mass is:

$$r_{cm}=\frac{M_{MW}\,r_{MW}+M_{31}\,r_{31}}{M_{MW}+M_{31}}$$

$$r_{cm}=\frac{(7\times10^{11}\,M_{sun})(0)+(10^{12}\,M_{sun})(690\,\text{kpc})}{7\times10^{11}\,M_{sun}+10^{12}\,M_{sun}}$$

$$r_{cm}=405\,\text{kpc}$$

This answer makes sense – M31 is a bit heaver than the Milky Way, so the center of mass should be closer to it than to us.

If the galaxies were orbiting each other circularly, then we could do the same thing we did for Problem 1 above. We could also just use Kepler's 3rd Law, measuring the period in years, the semimajor axis (our distance to the center of mass) in AU, and the mass in solar masses:

$$P^2(M_{tot})=a^3$$

$$P^2=\frac{\left((4.05\times10^5\,\text{pc})\times(2.0\times10^5\,\text{AU/pc})\right)^3}{1.7\times10^{12}\,M_{sun}}$$

$$P^2=3.43\times10^{20}\,\text{yr}^2$$

$$P=1.8\times10^{10}\,\text{yr}$$

Galaxies interact very slowly!

4) In a distant galaxy, an astronomer identifies a spectral line as being CaII (singly ionized Calcium), which has a rest wavelength of 393.3 nm. If in this galaxy, the wavelength is observed to by 410.0nm, then what would the equivalent recessional velocity be in km/sec, and what is the galaxy's redshift? Using a Hubble constant of 75 km/sec/Mpc, what is the distance to this galaxy?

The wavelength of the line hasn't changed too much, so just use nonrelativistic expressions:

$$z=\frac{\Delta\lambda}{\lambda}=\frac{16.7\,\text{nm}}{410.0\,\text{nm}}=0.0407$$

$$v=z\,c=0.0407\times(3\times10^5\,\text{km/s})=12200\,\text{km/s}$$

Using the Hubble law, we find the distance to be:

$$d=v/H_0=\frac{12200\,\text{km/s}}{75\,\text{km/s/Mpc}}=163\,\text{Mpc}$$

5) **The active galactic nucleus in the galaxy NGC 4151 is observed as basically a point source with a V-band magnitude of 12. If the Hα line (rest=656.3 nm) from the nucleus is observed at 658.5 nm, then what is the galaxy's distance (assume H_o=75 km/sec/Mpc)? What is the absolute magnitude of the nucleus (neglecting extinction)? How many times more luminous than the sun is the nucleus of NGC 4151 at opticaĺl wavelengths (V band)?**

To find the distance, we will again apply Hubble's Law; first, we'll need to find the recession velocity via its redshift:

$$v/c=\frac{\Delta\lambda}{\lambda}$$

$$v=\frac{2.2\text{ nm}}{656.3\text{ nm}}(3\times10^5\text{ km/s})=1006\text{ km/s}$$

$$d=v/H_0=\frac{1006\text{ km/s}}{75\text{ km/s/Mpc}}=13.4\text{ Mpc}$$

To find the absolute magnitude, we use the tried-and-true distance modulus formula:

$$m-M=5\log_{10}(d/10\text{ pc})$$

$$M=m-5\log_{10}(d/10\text{ pc})$$

$$M=12-5\log_{10}(1.34\times10^6)$$

$$M=-18.6$$

And to compare luminosities, we use the absolute magnitude/luminosity relation; the sun's absolute magnitude is about 4.8.

$$M_{AGN}-M_{sun}=-2.5\log_{10}(L_{AGN}/L_{sun})$$

$$L_{AGN}/L_{sun}=10^{\frac{M_{sun}-M_{AGN}}{2.5}}$$

$$L_{AGN}/L_{sun}=10^{\frac{4.8--18.6}{2.5}}$$

$$L_{AGN}=2.37\times10^9 L_{sun}$$