Spectral and Chemical Evolution of White Dwarf Stars

Doctoral Thesis Proposal

Eugene Chen

Department of Physics and Astronomy, UCLA

eugene@astro.ucla.edu

1. Introduction

White dwarf stars (WD hereafter) are known as the endpoint of stars whose initial mass is less than $6 \sim 8M_\odot$. Since the initial mass function of galaxies is in general dominated by stars of lower mass, it is important to have a thorough understanding of WDs. In fact, it is believed that over 97% of the stars in our galaxy will end up as a WD.

WDs are observed to possess mass around $0.6M_\odot$ and radius around $0.01R_\odot$ despite the wide range of their initial mass. The reason for this phenomenon remains unclear. Their compact nature results in high average density, large surface gravities and low luminosity, which makes them difficult to observe. The structure of such star is relatively simple: The main component is a large core of degenerate electrons and Carbon/Oxygen ions, with a thin, non-degenerate atmosphere. The atmosphere may be convective and/or radiative in nature, depending on various stellar parameters.

In contrast to the fact that WDs are very homogeneous in mass and size, they present wide variety in spectral appearances. In particular, it was shown that there exist certain "stellar gaps" in the cooling sequence. Such phenomenon can only be explained as an *evolutionary effect* rather than simply a reflection of the diversity in the underlying population. Many mechanisms such as accretion from Interstellar medium (ISM hereafter,) convective mixing and convective dredge up have been proposed. However, a detailed, realistic analysis which properly treat the subtle interplay between these mechanisms has never been accomplished. I propose to accomplish such goal in my thesis.

2. Observations and Analysis of White Dwarf Stars

As mentioned in the introduction, WDs present wide variety in spectral appearances. This fact is best reflected in the complicated spectrum classification system used by astronomers of this field. In order to state the observed phenomenon of WD spectral more efficiently, let me first introduce the current classification system, which is presented by Sion et al. (1983)

In the Sion system, each spectrum is assigned 3 to 5 symbols: (1) an uppercase D for degenerate; (2) an uppercase letter for primary spectroscopic type in the optical spectrum; (3) an uppercase

Table 1. The Sion System

DA	Only Balmar lines, No He I or metals present
DB	He I lines, no H or metals present
DC	Continuous Spectrum, no lines deeper than 5% in any part of the EM spectrum
DO	He II strong, He I or H present
DZ	Metal lines only, no H or He
DQ	Carbon feature, either atomic or molecular, in any part of the EM spectrum
Subscripts:	
P	polarization
wk	weak
e	emission
s	sharp
ss	very sharp
n	diffuseness
PEC	peculiar

Table 2. Examples

A WD with only HeI lines; $T_{eff} = 15000K$	DB3
A DB star with CaII; $T_{eff} = 14000K$	DBZ4
A polarized magnetic WD with H and He, but with He dominant;$T_{eff} = 20000K$	DBAP3

letter for secondary spectroscopic features, if present; and (4) a temperature index from 0 to 9 defined by $10\theta_{eff}(= 50400/T)$. Detailed definitions and examples are shown in table 1 and table 2.

Extensive observational work, best represented by that of Bergeron, Ruiz and Leggett (BRL hereafter) showed that the ratio of DA to non-DA WDs changes as a function of effective temperature along the cooling sequence. Most noteworthy, the cases of the existence of a so-called DB gap–an interval of effective temperature from 45,000K to about 30,000K in which no helium atmosphere object has been found–and of a cooler and narrower non-DA gap between $\sim 6000K$ and $\sim 5000K$ are well documented. Since stellar formation and evolution of different populations cannot be synchronized in advance, at least a significant portion of WDs have to switch their spectrum from H to He, back to H and then back to He again, during their lifetime.

More detailed investigation and analysis yields the following notable features:

1. For temperatures above 45000K, the DA far outnumbers the DO and down to 30000K the ratio continues to increase.

2. Some cool DAs between about $8000 \sim 11000K$ have a larger spectral line width while compare the spectral lines with hotter DAs.

3. While most cool white dwarfs classified as DA by the presence of Balmer lines stars have energy distributions fitting pure-H atmospheres, a minority fit atmospheres with $He \gg H$. These generally show Ca II and may have other lines due to heavy elements (spectral type DZA).

4. Conversely, while most cool non-DA stars (DC, DQ, DZ) have energy distributions fitting pure-He atmospheres, a minority fit pure-H models, even with the absence of H_α in the spectra.

5. A puzzling change in the DQ sequence of non-DA stars whose helium envelopes have dredged up carbon appears to occur at similar temperatures. While stars with C_2 bands in their optical spectra occur in the Bergeron samples down to about 6500K, below this temperature is where the peculiar DQ stars, which may show the C_2H molecule.

6. About 25% of the cool DAs show detectable Ca II lines in high resolution, high signal-to-noise ratio spectra-type DAZ. These exist over a wide Teff range, and parallel the helium-rich DZ/DZA types which may show broader lines formed in atmospheres at higher pressures.

7. Finally, in the last few years, some white dwarfs cooler than $T_{eff} < 4000K$ have been discovered. All appear to show strong deficiency in infrared wavelength.

Suggested mechanisms that could account for these phenomena will be explained in the following section. A detailed study that incorporates all of the mechanisms, however, is non-trivial due to some technical issues such as the effects of chemical mixing. We will come back to this point later.

3. Basic Mechanisms of Spectral Evolution

The Spectrum of a star is basically the photons that escape from within one optical depth. Therefore spectral evolution is equivalent to the evolution of state (chemical composition, temperature and pressure) of stellar photosphere. Here we examine some known mechanisms that govern this evolution.

3.1. Diffusion and settling

It is known that a WD is mainly composed of Carbon and Oxygen. However, the majority of WDs show hydrogen lines instead of Carbon/Oxygen lines. This is due to the effect of gravitational settling. As pointed out by Aller and Chapman (1960), the different responses of electrons and protons to gravity induced pressure and temperature gradients in a stellar atmosphere give rise to a small, but non-zero, electric field, which is necessary to maintain electrical neutrality. The electric, thermal and pressure forces do not cancel for minority ionic species whose charge to mass ratio differs from that of majority constituent, giving rise to a slow diffusion process. This process is particularly efficient in white dwarfs and leads to the formation of a pure envelope of the lightest element present on timescales $< 10^8$ years. The final compositional structure is an onion-skin like model, but the transitions are not strictly discontinuous, since there is an "ordinary" diffusion process associated with the chemical abundance gradient as well. In reality, each transition is marked by a diffusive tail even in the absence of other counteracting mechanism.

Gravitation settling has long been appreciated as the dominant factor of WD spectral evolution. The first phenomenon mentioned in section 2 can be naturally explained as a process towards equilibrium.

3.2. Radiative Levitation

The energy and momentum of a photon is related by the formula $E = pc$. Therefore, whenever the opacity is nonzero, material in the star will gain momentum from the photon. According to the calculation of Chayer et al. (1989), this effect could be of importance in DA stars with $T_{eff} > 20000K$ and non-DA stars with $T_{eff} > 30000K$. The equilibrium abundance of heavy elements in the atmosphere considering the effect on settling and levitation can be calculated precisely. Therefore, we can easily check this hypothesis. It turns out that in some cases, such as a WD named Feige 24, the metal abundance is perfectly predict by levitation theory. However, there also exists hot WDs without detectable traces of heavy elements and they definitely don't agree with levitation theory. Hence we concluded that this mechanism has a role in determining the spectrum but there are other mechanisms at play.

3.3. Convective dilution, dredge up and mixing

Convection occurs when a stratified liquid is unstable to adiabatic displacements of material between different layers. Mathematically this statement is formulated as the Schwarzschild criterion:

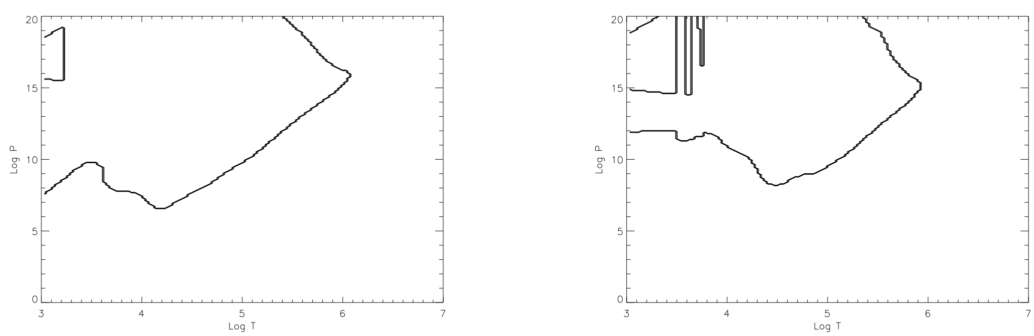
$$\frac{3}{16\pi acG}\frac{P\kappa}{T^4}\frac{L}{M} > \nabla_{ad} \tag{1}$$

where a stands for radiation density constant, c stands for speed of light, and κ stands for opacity.

Since the atmosphere is only a small portion of the white dwarf star, we can basically view L and M as a constant over the atmosphere. It is obvious that Schwarzschild criterion is most easily achieved when the opacity is large and/or the adiabatic temperature-pressure gradient is small. Therefore, we usually have a convection zone when the gas is in its partially ionized state.

A “phase diagram” which indicates the primary means of energy transport at a certain state (specified by T and P) can be made easily for a specific chemical composition of stellar atmosphere (given the L/M of that star). Figure 1(a) and Figure 1(b) are the phase diagrams for pure hydrogen and pure helium atmosphere of a WD with $\frac{L}{M} = 10^{-4}\frac{L_\odot}{M_\odot}$.

For a chemically stratified stellar atmosphere, we could have a hydrogen convection zone and/or a helium convection zone simultaneously/separately. As the convection zones evolve in their size and position, three significant events for spectral evolution may happen. The first scenario is termed convective dredge-up. It means the surface hydrogen convection zone becomes so big that it starts to engulf some of the underlying helium. As a result, a DA star may *gradually* turn into a DB star by this process. The scenario named convective dilution has a similar effect on spectral evolution. However, it refers to the case when the helium convection becomes big enough to engulf the covering hydrogen layer. Last but not least, convective mixing happens when the hydrogen convection zone and helium convection merge together. This will result in a *sudden* change of



(a) Hydrogen atmosphere

(b) Helium atmosphere

Fig. 1.— ”Phase diagram” of atmosphere for a WD with $L/M = 10^{-4}L_\odot/M_\odot$

surface chemical composition. (We can probably name it *super-dilution*)

3.4. Accretion from ISM

Since WDs are very compact objects and are bathed in ISM, we expect accretion could occur and place impacts on the spectrum. Indeed, the existence of cold, metallic WD is best explained by the effects of accretion since radiative levitation is not strong enough to maintain the observed metal abundance.

A description of such process is given by Bondi-Hoyle formula (1952):

$$\dot{M} = \frac{4\pi (GM)^2 \rho_\infty}{c_e^3} \tag{2}$$

Where $c_e^2 = c_\infty^2 + V^2$. c_∞ and ρ_∞ are the thermal speed and density at infinity and V is the relative speed between WD and local ISM. The dependence on velocity leads to interesting results. Since different components of the galaxy can be characterized by different kinetic properties, we could relate different types of spectral evolution to different components. Given the ISM density as $\rho \approx 2 \times 10^{-21}/cm^3$, a typical disk WD ($V \approx 30km/s$) should have an accretion rate about $\dot{M} \approx 5 \times 10^{-17} \frac{M_\odot}{yr}$ while a halo WD ($V \approx 150km/s$) might only have the accretion rate of $5 \times 10^{-20} \frac{M_\odot}{yr}$. (assuming it spends 10% of its lifetime in the disk)

Since the ISM is 70% hydrogen, 28% He and 2% heavy elements, the existence of cold, hydrogen-poor but metal-rich stars motivates us to find a mechanism which is able to prevent hydrogen from accreting. The most promising answer was given by Wesemael & Truran (1982), using the so-called "propeller mechanism" proposed by Illarionov and Sunyaev in 1975. However, this mechanism requires the ionization of hydrogen, so it is actually not very suitable for the case of cold stars.

3.5. Pressure Ionization

Since the opacity of hydrogen is much higher than that of helium, we normally associate the presence of hydrogen spectrum with the threshold hydrogen to helium ratio $\frac{M_H}{M_{He}} = 10^{-3}$. However, since the opacity of cold hydrogen mainly comes from the H^- ion, this threshold will be greatly reduced if the bound state which accounts for H^- is destroyed. Thus, pressure ionization sets a new relation between spectral evolution and the evolution of state of the stellar photosphere. This idea was being investigated by Malo, Wesemael and Bergeron (1999) to account for the existence of the "non-DA gap" at 5000K∼6000K. However, it seems that this effect isn't strong enough to explain the phenomenon completely.

4. The Goals, Current Status and Applications of the Research

In my thesis I plan to make a systematic incorporation of all the mechanisms indicated in the last section, with the potential possibility of including other mechanisms. We are expecting a most realistic simulation available at the final stage.

As the first step towards this goal, we have constructed a toy accretion model for WDs and an adiabatic gradient calculator for mixtures of Hydrogen and Helium.

4.1. Toy model for ISM accretion

Here we estimate the effects of ISM accretion on WD spectrum without considering the change of opacity and adiabatic gradient while the envelope is being polluted by ISM. (Namely, the structures of the stars are determined only by its effective temperature and its original composition–to be specific; we consider the case of pure helium.) We assume that the chemical composition of photosphere is homogeneously mixed with the surface convection zone. Thus, a DB with a big convection zone will evolve into a DA slower than the one with a smaller convection zone. (Given the same size of photosphere and the same accretion rate.) The envelope structure was calculated by the standard procedure described in Hansen, Kawaler and Trimble (2004). The pressure at the base of the photosphere is found by first evaluating the integral $\tau = \int \frac{g}{\kappa} dP$ (κ is the Rosseland mean opacity) numerically then shoot the root of $\tau = 1$. We adopt opacity from OPAL group (1992) and equation of state/adiabatic gradient from Saumon, Chabrier, and Van Horn (SCVH hereafter)(1995).

Figure 2 shows the location of convection zone in pressure space as a function of effective temperature. Since the radiative zone between photosphere and convection zone is very small, if any, our assumption about chemical homogeneity is likely to be valid.

We further assume a constant accretion rate, (i.e., the star sits in an environment with constant density of ISM over space and time,) thus the amount of accreted ISM is proportional to the cooling time of that star. We can estimate this cooling time as a function of current effective temperature by the theory given by Mestel(1952), which states that $\tau_{cooling} \propto (\frac{L}{M})^{-5/7}$. In Figure 3 we present the mass of accreted hydrogen and helium as a function of T_{eff}, respectively. A brief introduction to Mestel's cooling theory is given in the appendix.

Quoting the accretion rate of a typical disk WD, we have $M_{accreted} = 5 \times 10^{-17} \tau_{cooling}$. Thus, we can find the mass ratio between hydrogen and helium in photosphere as a function of effective temperature. The result is shown in Figure 4, indicated by the solid line. We have assumed that ISM is 70% hydrogen and 28% helium in our calculation.

In the above simulation, we've totally ignored the fact that, as the ISM accretes, the opacity/adiabatic gradient will be gradually approaching to that of pure hydrogen.(Due to both accretion

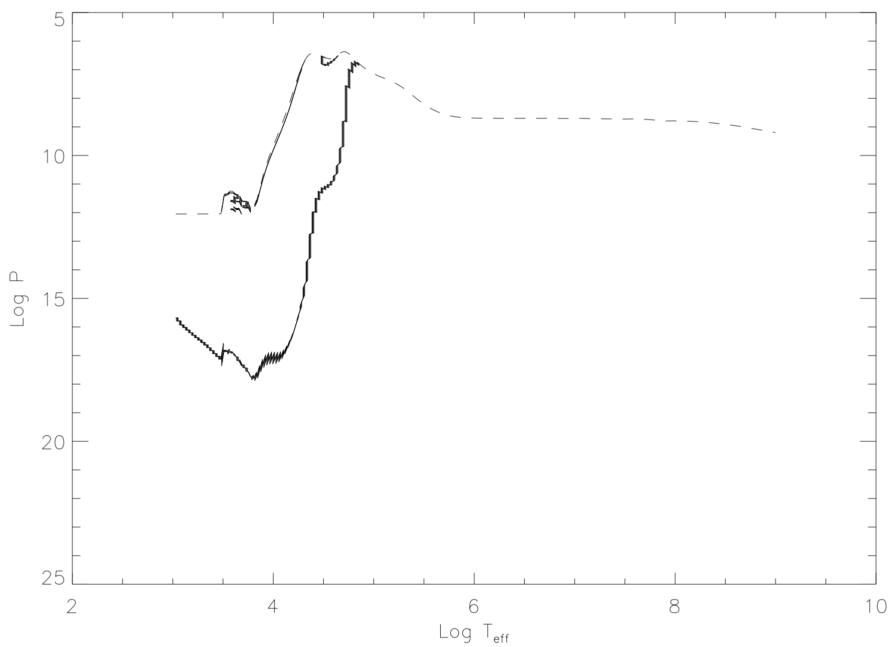


Fig. 2.— The location of convection zone(s) in pressure space as a function of effective temperature in a pure Helium atmosphere. The base of photosphere is indicated by doted line. The solid line indicates transitions between radiative and convective zones. We've adopted $M_{WD} = 0.6M_{\odot}$ in our model.

and diffusion) The best way to get the problem around is probably to divide the whole process into many small intervals and recalculate the opacity and equation of state at each step. In fact, we have constructed an adiabatic gradient calculator suitable for this purpose. However, we can make a rough investigation by doing the same simulation for a pure hydrogen atmosphere. As long as H and He are the only relevant elements, the true outcome should lie somewhere between the two limit cases. The result for the case of hydrogen is indicated by a dotted line in Figure 4.

At this point, two facts might be worth mentioning. First of all, we are showing a sudden $\frac{M_H}{M_{He}}$ increase at $\log T_{eff} \approx 3.5 \sim 3.7$ whereas the non-DA gap described in BRL(1997) is located at $\log T_{eff} \approx 3.7 \sim 3.8$ (as indicated by the verticle lines in figure 4.) Secondly, if the threshold M_H to M_{He} ratio for the appearance of Balmer lines is the generally accepted value 10^{-3}, then we will not be able to observe any non-DA stars.

Since we are just presenting a toy model, the first fact suggests that we are on the correct direction towards the explaination of the non-DA gap. The second fact sounds a little embarrassing, however, Koester (1976) has suggested that the formula we used for accretion rate (i.e., equation 2) is an overestimate. Therefore, with some careful analysis, we should be able to give a nice account on this issue.

4.2. The adiabatic gradient calculator

We further investigate the effects of mixing hydrogen and helium, following SCVH(1995).

We do interpolations of the equations of state (between H and He) by the so called "additive-volume rule," which states that every specific extensive variable $W(P,T)$ of the mixed system is given by:

$$W(P,T) = \sum_i X_i W^i(P,T) \tag{3}$$

Where X_i is the fraction of the total system occupied by subsystem i. By using this rule, we are neglecting all the interactions between H and He, hence we can not account for the physics of chemical equilibrium and immiscibility. This implies that, at the regime where these factors are important, (e.g., the regime of partial ionization) the rule is probably not very reliable. Nonetheless, we will stick to it just for its simplicity.

As an example of this rule, since density is the inverse of specific volume, which is an extensive variable, we can interpolate the density of a H-He mixture with helium weight fraction Y by:

$$\frac{1}{\rho(P,T)} = \frac{1-Y}{\rho^H(P,T)} + \frac{Y}{\rho^{He}(P,T)} \tag{4}$$

However, when we are calculating the entropy, an additional "entropy of mixing" term should be included to recover the ideal gas limit.

$$S(P,T) = (1-Y)S^H(P,T) + YS^{He}(P,T) + S_{mix}(P,T) \tag{5}$$

We are concerned about the adiabatic gradient while calculating the structure of atmosphere since this single quantity determines the RHS of Schwarzschild criterion. By definition and an identity in differential calculus:

$$\nabla_{ad} = \frac{\partial \log T}{\partial \log P}|_S = -\frac{\frac{\partial \log S}{\partial \log P}|_T}{\frac{\partial \log S}{\partial \log T}|_P} \tag{6}$$

The ideal entropy of mixing of m system each containing N_i particles is:

$$\frac{S_{mix}}{k_B} = N \ln N - \sum_{i=1}^{m} N_i \ln N_i \tag{7}$$

where

$$N = \sum_{i=1}^{m} N_i \tag{8}$$

Therefore, when combining a pure hydrogen subsystem containing $\mathcal{N}_H$ particles (of all hydrogen species, including the ionized electrons) and a pure helium subsystem of $\mathcal{N}_{He}$ particles, we have a total entropy of mixing as:

$$\begin{aligned} \frac{S_{mix}}{k_b} &= (\mathcal{N}_H + \mathcal{N}_{He}) \ln(\mathcal{N}_H + \mathcal{N}_{He}) \\ &- N_{H_2} \ln N_{H_2} - N_{H^+} \ln N_{H^+} - N_e \ln N_e \\ &- N_{He} \ln N_{He} - N_{He^+} \ln N_{He^+} - N_{He^{2+}} \ln N_{He^{2+}} \end{aligned} \tag{9}$$

where N_e is the total number of electrons. We can figure out all of the quantities given in the right hand side of the above formula from the EOS provided by SCVH. Thus, S_{mix} can be calculated. Figure 5 is a contour plot of the entropy of mixing and Figure 6 is the total entropy for 0.75grams of Hydrogen and 0.25grams of Helium. We use the Boltzmann constant as our nature unit.

Now we can find the adiabatic gradient of a H-He mixture by using (6), the results are shown in Figure 7 for Y=0.25.

4.3. Future and Applications of the Research

One immediate thing to continue on this research might be the incorporation of Sec 4.1 and Sec 4.2, as already mentioned. The simulation could become subtle when we are considering the accretion of metal, due to the fact that 1. metals will be driven downwards into the stellar interior via gravitational settling and 2. the opacity of atmosphere is sensitive to metallicity. The situation is further complicated when we consider a non-uniform accretion rate. Such situations could raise naturally in the two-phase model of ISM. Since more timescales are involved, non-equilibrium phenomena could emerge. At the final stage of this research, we expect to accomplish (at least) a full, detailed and realistic simulation on every process mentioned.

The most practical application of our research is probably on the estimation of the ages of stellar populations. Since white dwarf stars have a relatively small mass dispersion, we can infer that dimmer white dwarf stars cools down slower. Therefore, if we construct a luminosity function by making a survey of white dwarf luminosity distribution in a certain stellar population (e.g. a globular cluster), we would expect that the function increases as the luminosity gets lower but will have a sudden cutoff due to the finite age. A theory on WD cooling is needed for reading out the age from this cutoff. The Mestel model (see appendix) is not accurate since it makes the assumption of a radiative and fully ionized atmosphere throughout its lifetime. This is of course far from truth, otherwise we won't be able to detect any spectral lines and be proposing research plan on spectral evolution.

Hopefully, with the deeper understanding on spectral evolution brought by the proposed study, we will be able to feed in the theory of WD cooling with substantial improvements.

A. Mestel's Theory of WD cooling

The first modern theory on white dwarf cooling was given by L. Mestel in 1952. Although a lot of details can be modified and improved, the general principles survive largely intact.

Mestel pictured the core of white dwarf as isothermal since degenerate electron has a very high conductivity. He also assumed the surface was made of free electrons–so Kramer's opacity could be used. The cooling rate is mainly determined by the radiation diffusion at the surface since it is the "bottleneck" process.

From a thermodynamics point of view, luminosity is proportional to negative the rate of change of internal entropy:

$$T\frac{ds}{dt} = C_v\frac{\partial T}{\partial t} - \frac{T}{\rho^2}(\frac{\partial P}{\partial T})_\rho\frac{\partial \rho}{\partial t} \tag{A1}$$

In Mestel's model, we assume that there is no change in volume, so we have:

$$L = -C_v \frac{\partial T_{core}}{\partial t} = -\frac{3}{2} \frac{kM}{Am_p} \frac{\partial T_{core}}{\partial t} \tag{A2}$$

The second equality follows from the kinetic theory of ideal gas since the atmosphere is non-degenerate.

On the other hand, luminosity is also a function of surface area and temperature gradient:

$$\frac{dT}{dr} = -\frac{3}{4ac} \frac{\kappa\rho}{T^3} \frac{L}{4\pi r^2} \tag{A3}$$

Hydrostatic requires:

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} \tag{A4}$$

With the assumption of Kramer's opacity we then have

$$\frac{dP}{dT} \propto \frac{T^{6.5}}{\rho L} \tag{A5}$$

Near the surface, an ideal gas equation of state is not bad, therefore we can infer $\rho \propto \frac{P}{T}$, yielding

$$PdP \propto \frac{T^{7.5}}{L} dT \tag{A6}$$

Integration gives

$$P^2 \propto \frac{T^{8.5}}{L} \Rightarrow L \propto \frac{T^{8.5}}{P^2} \tag{A7}$$

To tell the luminosity of a white dwarf star, you measure T and P separately at an *arbitrary* point in the atmosphere, then plug them into the above formula.

However, we know the ratio $T^{8.5}/P^2$ has to be determined by T_{core} since luminosity and T_{core} should have a one to one correspondence. (Hot things always has a higher luminosity) What is the relation?

Well, given a T_{core}, we can find the pressure of the atmosphere base by noticing that at the base of the atmosphere, 1.temperature is *roughly* equal to that of the core and 2. ideal gas law and degenerate pressure formula both *roughly* apply. Therefore, we set:

$$\begin{aligned} T_{base} &= T_{core} \\ \Rightarrow \rho_{base} T_{base} = \rho_{base} T_{core} &\propto \rho_{base}^{5/3} \\ \Rightarrow \rho_{base} &\propto T_{core}^{3/2} \end{aligned} \tag{A8}$$

$$\Rightarrow P_{base} \propto T_{core}^{5/2} \tag{A9}$$

The last line follows from ideal gas law. Luminosity can thus be evaluate by $L \propto (T^{8.5}/P^2)_{anywhere\ of\ atmosphere}$ $= (T^{8.5}/P^2)_{base} \propto T_{core}^{3.5}$

That is, a higher core temperature makes the non-degenerate layer thicker thus produce a greater discrepancy between the effective temperature and core temperature. Therefore $L \sim\ T^{3.5}$ instead of T^4 as the case for blackbody. When we combine the thermodynamics formula (A2) and the above formula, we get luminosity as a function of mass and time:

$$L \propto M\ t^{-7/5} \tag{A10}$$

This is the cooling theory we used in our toy model for estimating the total amount of accreted material.

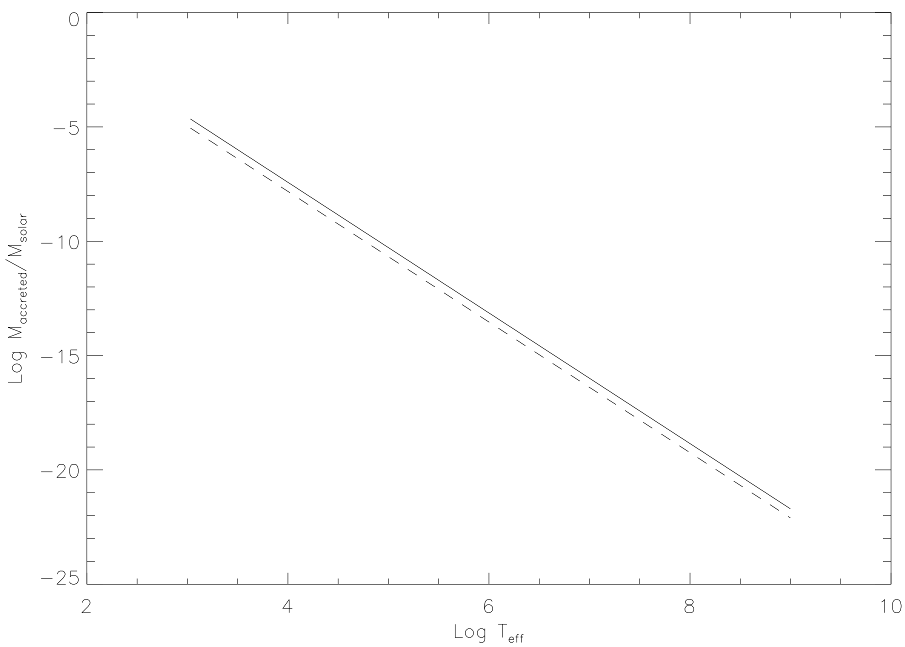


Fig. 3.— Accreted hydrogen mass (solid line) and accreted helium mass (dotted line) as a function of T_{eff} in log-log coordinates.

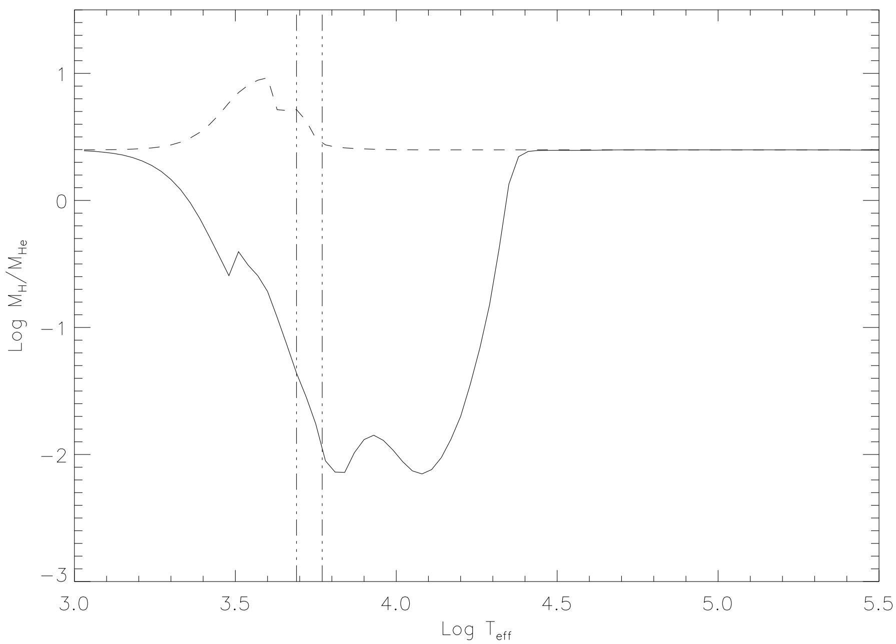


Fig. 4.— The evolution of $\log \frac{M_H}{M_{He}}$ due to ISM accretion. The solid line indicates the case of an initially pure helium atmosphre while the dotted line indicates the case for an initially pure hydrogen atmosphere. The non-DA gap described by BRL(1997) is indicated by two vertical lines.

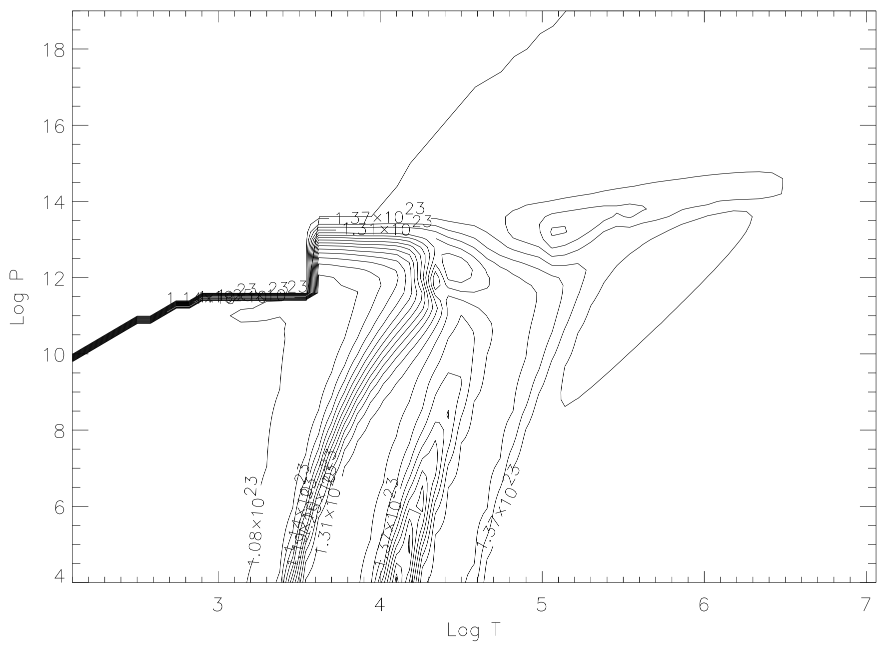


Fig. 5.— Entropy of mixing for 1 gram of $Y = 0.25$ hydrogen-helium mixture.

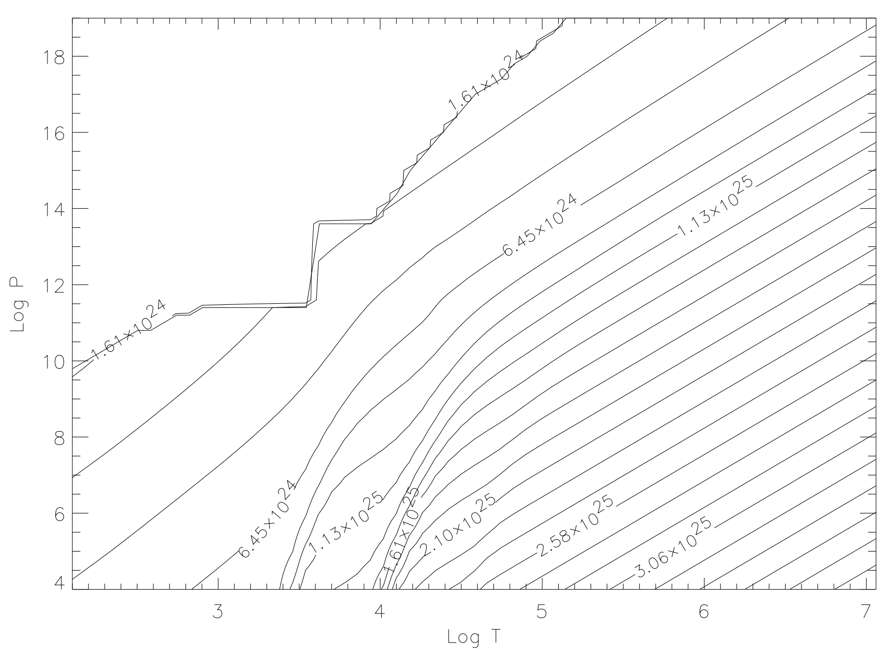


Fig. 6.— Specific entropy for Y=0.25 hydrogen-helium mixture.

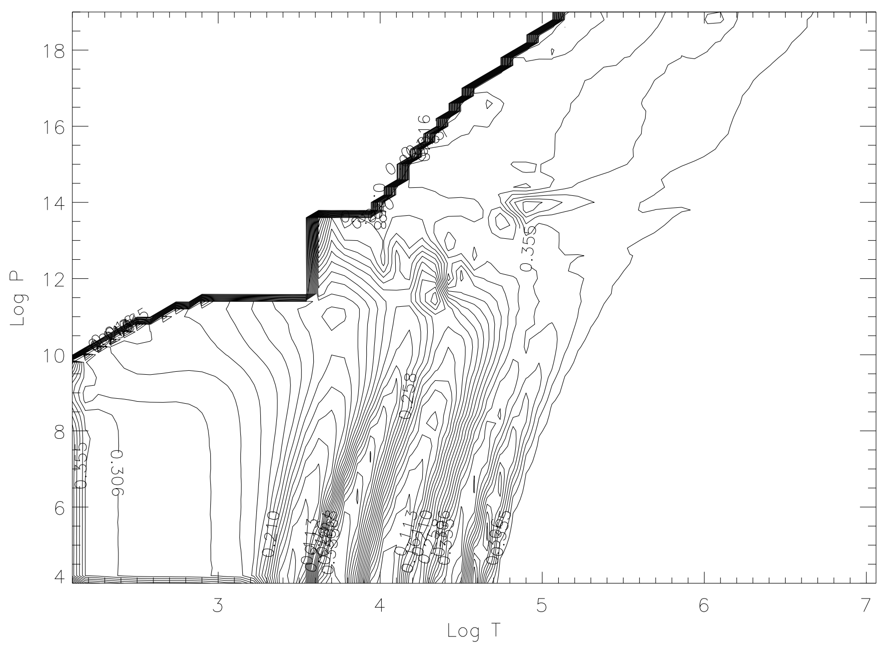


Fig. 7.— Adiabatic gradients for $Y = 0.25$ hydrogen-helium mixture.