CMB Spectrum

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History: CMB Observations

• 1940: CN R(1) line observed toward ζ Oph by McKellar and ignored by cosmologists*.  
• 1946: Dicke, Beringer, Kyhl & Vane (PR, 70, 340-348), *Atmospheric Absorption Measurements with a Microwave Radiometer*: “there is very little (< 20° K) radiation from cosmic matter at [1-1.5 cm] wave-lengths.”  
• 1957: Denisse, Lequeux & Le Roux (CR, 244, 3030-3033): $T_{\text{ciel}} < 3 \text{ K at } \lambda = 33 \text{ cm}$. 
On the CN non-discovery


Herzberg (1950) in Spectra of Diatomic Molecules, p 496:

“From the intensity ratio of the lines with K=0 and K=1 a rotational temperature of 2.3° K follows, which has of course only a very restricted meaning.”

There went Herzberg’s [second] Nobel Prize.
Fred Hoyle missed the Nobel Prize

• Hoyle (1950), reviewing a book by Gamow & Critchfield: “[the Big Bang model] would lead to a temperature of the radiation at present maintained throughout the whole of space much greater than McKellar's determination for some regions within the Galaxy.”

• This book implied $T_o = 11$ K. Gamow in 1956 Scientific American implied 6 K. Alpher & Herman explicitly gave 5 K or 1 K.

• Nobody followed this up!
CN followup after Penzias & Wilson

• Reworking and reobserving the CN lines gave $2.78 \pm 0.10$ K at 2.64 mm. (Thaddeus, 1972, ARAA, 10, 305-334)

• By 1993, $2.73 \pm 0.03$ K
Atmospheric Absorption Measurements with a Microwave Radiometer

Robert H. Dicke, Robert Beringer, Robert L. Kyhl, and A. B. Vane
Massachusetts Institute of Technology, Cambridge Massachusetts
(Received May 18, 1946)

The absorption of microwave radiation in traversing the earth’s atmosphere has been measured at three wave-lengths (1.00 cm, 1.25 cm, and 1.50 cm) in the region of a water-vapor absorption line. The measurement employs a sensitive radiometer to detect thermal radiation from the absorbing atmosphere. The theory of such measurements and the connection between absorption and thermal radiation are presented. The measured absorption together with water-vapor soundings of the atmosphere permits the calculation of the absorption coefficients at standard conditions (293°K, 1015 millibar). These are 0.011, 0.026, and 0.014 db/km/g H₂O/m³ for the wave-lengths 1.00 cm, 1.25 cm, and 1.50 cm, respectively. These values are (50 percent) greater than those given by the theory of Van Vleck. The collision width of the line and its location are in better agreement with the theory and infra-red absorption measurement. It is also found that there is very little (<20°K) radiation from cosmic matter at the radiometer wave-lengths.
On Dicke’s non-discovery

• Dicke was reporting on war-time work done to see if K band radar was practical. The atmospheric absorption was low enough.

• Dicke invented the differential radiometer for this work. This compares a source to a reference source. The switch used to connect the two sources alternately to the radiometer is called a Dicke switch.

• Dicke had all the tools needed to measure the 3 K CMB in 1945.
Dicke’s Antenna

A Flared Horn for low sidelobes

Fig. 2. Tapered, rectangular, horn antenna.
Smooth Transition to Free Space

• Dicke’s flared horn works and was emulated by FIRAS since FIRAS has a 100:1 wavelength range.
• Modern anisotropy experiments use corrugated horns. The grooves act like shorted $\lambda/4$ stubs which act like open circuits so modes in the horn are very nearly TEM free-space modes.
Dicke lacked a matched load

• Dicke needed only a low temperature load
• Using a room temperature reference load, the difference signal at the zenith was 250 K
• While the sky signal was very nearly equal to the expected atmospheric emission at the zenith, the large chopped signal limited the experimental accuracy to 20 K.
• If Dicke had known that the CMB might exist, he could have measured it except for the small problem of World War II.
On the French non-discovery

• Le Roux in his unpublished thesis gave $T_{\text{ciel}} = 3 \pm 2 \text{ K}$.

• But the receiver had a noise temperature of 1450 K, and worked in total power. It did not use a Dicke switch.

• The dish antenna used would give 100 K signal from the ground in the backlobes.

• The paper published in Comptes Rendus gives $T_{\text{ciel}} < 3 \text{ K}$ and $\Delta T < 0.5 \text{ K}$. Both are wrong at $\lambda = 33 \text{ cm}$ or 0.9 GHz.
On the Bell Labs horn

• Designed for very small sidelobes and negligible backlobes.
• Used for detecting signals bounced off the Echo satellites – big passive aluminum coated Mylar balloons in low Earth orbit.
• Ohm (1961, BLTJ, 40, 1065-1094) reported a total $T_{sys} = 22.2 \pm 2.2$ K vs $18.9 \pm 3$ K predicted at 2.4 GHz. The errors are very conservative. This is a detection that was ignored.
Discovery of the Cosmic Microwave Background
On Penzias & Wilson

• They were not satisfied with an unexplained 3.5 K excess noise temperature at 4.08 GHz and went to great lengths to chase it down, including removing the “white dielectric material” left by pigeons nesting in the horn.
• They used a Dicke radiometer with a liquid helium cooled load. So did Ohm.
• They spoke to Bernie Burke who actually knew that Dicke was building a radiometer to look for a signal just like this.
History of the Spectrum

• Many ground-based and mountain-top measurements filled in the 0.3-20 cm wavelength range, giving $T = 2.73 \pm 0.08$ K.

• Reworking and reobserving the CN lines gave $2.78 \pm 0.10$ K at 2.64 mm. (Thaddeus, 1972, ARAA, 10, 305-334)

• Big excesses over blackbody seen or not seen by different rocket and balloon experiments.
Plenty of nonsense to go around

- 2000 MJy/sr excess at 0.8 mm seen by Houck & Harwit (1969, ApJL, 157, L45)
- No excess seen by MIT group (Muehlner & Weiss 1972)
Woody & Richards 2 mm excess

![Graphs showing frequency vs. detector response and flux vs. frequency with various lines and markers indicating resolution, optical, and 2.96 K Blackbody.](image)
The Good, Bad & Ugly fits to WR

• Good fit: \( \varepsilon B_{\nu}(T=2.73 \text{ K}) \), the calibration error model

• Bad fit: Jakobsen, Kon & Segal (1979, PRL, 42, 1788) angular momentum model

• Ugly fit: interstellar dust at \( z=100 \)
What went wrong with WR

• Load was a blackbody at the helium bath temperature of 1.7 K. There was large amplitude central maximum signal from the 2.7 K CMB which required an accurate calibration.

• Calibration was based on atmospheric oxygen lines which gave small wiggles throughout the interferogram

• ADC differential non-linearity affected the calibration [John Mather’s hypothesis]
Global linearity can be good
ADC Differential Nonlinearity 2

But a small region like the blue box
ADC Differential Nonlinearity 3

can have a slope quite different from the global slope.
Another Rocket Excess

  - This is the Berkeley-Nagoya experiment
  - Includes Paul Richards
- $T_B = 2.80$ K at 1.1 mm; 2.96 K at 0.7 mm & 3.18 K at 0.5 mm.
- This one didn’t last long. COBE launched in 1989.
What went wrong with BN?

- Probably some optical element heated to 4 K by the vibrations of launch.
- Cooldown by radiation at 3 K is very slow.
- FIRAS saw a very small excess emission from some warm thing that decayed with a 55 day time constant. The team called it the “Unidentified FIRAS Object”. In a 5 minute rocket flight this would look cosmic.
Original COBE was for a Delta
COBE was directed to use the Shuttle

- This picture is the shuttle launched version of COBE which was actually nearly completed in Jan 1986.
- Then the Challenger blew up on launch…
Challenger, STS-51

Because of the Challenger disaster, NASA dropped plans to fly the shuttle from Vandenberg AFB in California, so COBE’s polar orbit required a new ride.
The Delta DIET

• The shuttle version of COBE weighed 5,000 kg and also needed a 700 kg vacuum pump in the shuttle bay.
• It was the full shuttle payload from Vandenberg AFB. A > 500 M$ launch.
• COBE had to be redesigned to fit on a Delta.
• The mass went down to 2300 kg.
• The launch cost went down to about 30 M$.
• No science was lost, but the schedule took a 2 year hit.
This is the final Delta version of COBE
FIRAS on COBE

- Built using lessons learned in Woody & Richards. [Mather was an earlier graduate student working on this experiment.]
  - Internal reference source kept at $T_0$ instead of 1.7 K bath.
  - Baseline dithered using a random number generator and a DAC for each scan
  - Very symmetric design.
  - External calibrator black to 0.99999 or better.
FIRAS measured the CMB spectrum

- Far InfraRed Absolute Spectrophotometer
- A differential polarizing Michelson interferometer
- Zero output when the two inputs are equal
- One input is either the sky or a very good blackbody, other is a pretty good blackbody
Polarizing Michelson

Output is vertical, left circular, horizontal, right circular
Fourier Transform Spectroscopy

[Graphs and diagrams showing Fourier Transform concepts]
Personal History: my FIRAS breadboard at MIT

Water lines @ 19 & 25 cm\(^{-1}\) in lab air
Diagram of FIRAS
Differential Spectroscopy

![Graph showing interferogram with different lines labeled XCAL@2.750 - ICAL@2.759, SKY - ICAL@2.759, XCAL@2.733 - ICAL@2.759, XCAL@2.701 - ICAL@2.759. The x-axis is labeled as x[cm].]
FIRAS Final Spectrum

- SZ Effect $\propto$
  $y = N_e \sigma_T kT_e / m_e c^2 < 15 \times 10^{-6}$
- Bose-Einstein
  $\mu < 9 \times 10^{-5}$
- Energy from hot electrons into CMB
  $< 60$ parts per million
Residual is the Measurement

- FIRAS measures \( R_\nu = I_\nu - B_\nu(T_{\text{cal}}) \)
- Data analysis subtracts off a temperature change and a galactic signal:
  \[- \Delta I_\nu = R_\nu - (T_0 - T_{\text{cal}}) \frac{dB_\nu}{dT} - G g(\nu) \]
- Weighted RMS of \( \Delta I_\nu \) is only 20 kJy/sr.
- Thus anything added to the CMB must either be quite small, or have a spectrum like either \( \frac{dB_\nu}{dT} \) or \( g(\nu) \).
FIRAS Far IR Background

• Any proposed FIR background must be compatible with the limits on the CMB distortion.

• This means the FIRB is either small or similar to the galactic spectrum.

• The FIRAS fit to the $N_H = 0$ intercept of the high frequency channel, [astro-ph/9803021]

$$I_{\nu} = 1.3 \times 10^{-5} (\nu/100)^{0.64} B_{\nu}(18.5 \text{ K}),$$

fails with $\Delta \chi^2 = 22.6$ (4.75 $\sigma$)
No simple limit at 850 μm

- FIRAS fit fails with $\Delta \chi^2 = 22.6$ but $I_{850} = 143$ kJy/sr
- Lagache et al. fit [astro-ph/9901059] is marginal with $\Delta \chi^2 = 5.8$ but $I_{850} = 115$ kJy/sr
- The modified scaled Primack model on the next slide is fine with $\Delta \chi^2 = 2.2$ but $I_{850} = 195$ kJy/sr
Cosmic Optical & IR Background

![Graph showing mean intensity vs frequency and wavelength.](image-url)
Other data plus FIRAS
How did the CMB get so black?

- A blackbody must be opaque and isothermal.
A star is not isothermal

- Visible temperature gradient is necessary for radiative transfer.
Gray atmosphere
Mixture of T’s look like a y

- \( N \propto T^3 \) for a blackbody
- \( U \propto T^4 \) for a blackbody
- \( U/N^{4/3} \propto \text{const} \) for a blackbody
- For 50:50 mixture of \( T(1-\varepsilon) \) and \( T(1+\varepsilon) \)
  - \( N \propto T^3[(1-\varepsilon)^3+(1+\varepsilon)^3]/2 = T^3(1+3\varepsilon^2) \)
  - \( U \propto T^4[(1-\varepsilon)^4+(1+\varepsilon)^4]/2 = T^4(1+6\varepsilon^2+\ldots) \)
  - \( U/N^{4/3} \propto 1+2\varepsilon^2 = 1+4y \)
- NOT a BLACKBODY!
- So effective \( y = 0.5\varepsilon^2 = 0.5 \) variance(\( \Delta T/T \))
Gray Atmosphere looks like $y = 0.0062$
Cosmological $T$ vs redshift
Redshift cancels $T(z) \propto (1+z)$

- Apparent temperature of radiation from all redshift shells is the same.
Non-Standard Models Fail

• Cold Big Bang: Layzer & Hively (1973) need a very large opacity during and after the Pop III starburst. Needle-shaped grains? Wright (1982)
$T/(1+z)$ is not constant
Cold Big Bang Energy Problem

- Comoving CMB energy density is 0.25(1+z) eV/cc.
- Comoving baryon rest mass density is 10 \( \Omega_B h^2 \) keV/cc.
- Conversion of fraction \( \Delta X \) of baryons from H to He yields only 70 \( \Omega_B h^2 \Delta X \) eV/cc.
- We know \( \Delta X < 0.04 \) and we know without using the CMB or BBNS that \( \Omega_B h^2 \) is small.
- Hence (1+z) = 0.07(\( \Delta X / 0.04 \))(\( \Omega_B h^2 / 0.022 \)) has to be small, we need too many baryons, and a high dust density is needed to be opaque at small redshift.
- But Universe can’t be opaque at small redshift because we see mm-wave CO lines from QSOs at \( z \approx 5 \).
Steady State Model Cartoon
Steady State Model Fails

• $T(z) = T_o$ in the Steady State.
• $T_{\text{app}} = \frac{T(z)}{1+z} = \frac{T_o}{1+z}$ is not constant.
• Wide range of $T_{\text{app}}$’s leads to a spectrum with large effective Kompaneets $y$.
• For a gray opacity with optical depth $\tau$ per Hubble radius, $y_{\text{eff}} = \frac{1}{2}\tau^2$ so $\tau > 180$ is needed.
• Universe can not be that opaque now because we see millimeter CO lines from distant QSOs.
Tired Light Model Fails

- For expanding models:
  - $E/\text{photon} \propto T \propto (1+z)$
  - $N \propto (1+z)^3$
  - $U \propto (1+z)^4$
  - $U/N^{4/3} \propto \text{const}$

- The tired light models:
  - $E/\text{photon} \propto T \propto (1+z)$
  - $N \propto \text{const}$
  - $U/N^{4/3} \n \not\propto \text{constant}$

- Precise Tolman surface-brightness test
X-Ray Background

Hot diffuse plasma gives right spectrum but violates FIRAS limit on $y$. Source thought to be highly obscured AGNs.
Sunyaev-Zeldovich Effect

Hot electrons increase high $\nu$'s, decrease low $\nu$'s
Distortions by electrons

Kompaneets (1957, Sov. Phys. JETP, 4, 730) equation:

\[
\frac{\partial n}{\partial y} = x^{-2} \frac{\partial}{\partial x} \left[ x^4 \left( n + n^2 + \frac{\partial n}{\partial x} \right) \right]
\]

where \( n \) is the number of photons per mode (\( n = 1/(e^x - 1) \) for a blackbody), \( x = h\nu/kT_e \), and the Kompaneets \( y \) is defined by

\[
dy = \frac{kT_e}{m_e c^2 n_e \sigma_T c dt}.
\]

Total photon number conserved:

\[
\frac{\partial N}{\partial y} \propto \int x^2 \frac{\partial n}{\partial y} \, dx
\]

\[
= \int \frac{\partial}{\partial x} \left[ x^4 \left( n + n^2 + \frac{\partial n}{\partial x} \right) \right] \, dx
\]

\[
= 0
\]
Drive to Bose-Einstein for t < 75 yrs

The stationary solutions $\partial n/\partial y = 0$ are general Bose-Einstein thermal distributions:

$$n = 1/(\exp(x + \mu) - 1)$$

When $z > z_y$,

$$(1 + z) \frac{\partial y}{\partial z} = \sigma_T n_{e,0} \frac{kT_o}{m_e c^2} \frac{c}{H} (1 + z)^4 > 1$$

$$z_y = 10^{5.0} / \sqrt{\Omega_B h^2 / 0.0224}$$
Small-y limit

Start with blackbody at $T_\gamma$.
Scatter off electrons with $T_e$.
Let $f = T_e / T_\gamma$
$n_\circ = 1 / (\exp(fx) - 1)$. Therefore

$$
\left( n + n^2 + \frac{\partial n}{\partial x} \right) = \frac{1}{\exp(fx) - 1} + \frac{1}{(\exp(fx) - 1)^2} - \frac{f \exp(fx)}{(\exp(fx) - 1)^2}
$$

$$
= \frac{(\exp(fx) - 1 + 1 - f \exp(fx))}{(\exp(fx) - 1)^2}
$$

$$
= \frac{(1 - f) \exp(fx)}{(\exp(fx) - 1)^2}
$$

$$
= (1 - f^{-1}) \frac{\partial n}{\partial x}
$$
Standard S-Z spectrum

Defining the “distorting” $y_D$ as

$$dy_D = \frac{k(T_e - T_\gamma)}{m_e c^2} n_e \sigma_T c dt$$

$$T_\nu = T_\circ \left[1 + y_D \left( \frac{x(e^x + 1)}{e^x - 1} - 4 \right) + \ldots \right].$$

where $x = h \nu / k T_\circ$. 
Energy Transfer for $y$

Energy transfer: $U \propto \int x^3 n \, dx$ so

$$\frac{\partial U}{\partial y_D} = \int x \frac{\partial}{\partial x} \left( x^4 \frac{\partial n}{\partial x} \right) \, dx$$

$$\frac{\partial U}{\partial y_D} = -\int \left( x^4 \frac{\partial n}{\partial x} \right) \, dx$$

$$= 4 \int x^3 n \, dx = 4U$$

FIRAS limit $y_D < 1.5 \times 10^{-5}$ implies

$$\Delta U/U < 6 \times 10^{-5}$$
N and U vs μ for Bose-Einstein

\[ N \propto \int \frac{x^2 dx}{\exp(x + \mu) - 1} \]
\[ = \sum_{k=1}^{\infty} e^{-k\mu} \int x^2 e^{-k x} dx \]
\[ = 2 \sum_{k=1}^{\infty} \frac{e^{-k\mu}}{k^3} \]
\[ = 2 (\zeta(3) - \mu \zeta(2) + \ldots) \]

A similar calculation for the energy density shows that

\[ U \propto 6 (\zeta(4) - \mu \zeta(3) + \ldots). \]

For \( N = \text{const} \), need \( \Delta T/T = \mu \zeta(2)/(3\zeta(3)) \).
Energy transfer for $\mu$

Therefore, the energy density change at constant $N$ is

$$\frac{\Delta U}{U} = \left( \frac{4\zeta(2)}{3\zeta(3)} - \frac{\zeta(3)}{\zeta(4)} \right) \mu = 0.714\mu.$$  

FIRAS limit $|\mu| < 9 \times 10^{-5}$ implies

$$\Delta U/U < 6 \times 10^{-5}$$
Photon Sinks & Sources

- Bremsstrahlung requires an electron and a proton, which are both very rare.
- Double photon Compton scattering only requires one rare electron.
Final blackening

Double photon Compton scattering

\[ \Delta p \approx h\nu/c \]

\[ a \propto h\nu^2/(m_e c) \quad \text{for} \quad \Delta t \propto 1/\nu \]

The energy radiated in new photons is thus

\[ P \Delta t \propto \frac{2e^2a^2}{3c^3} \Delta t \propto e^2h^2\nu^3/(m_e^2c^5) \]

New photons per scattering

\[ \frac{P \Delta t}{h\nu} \propto \frac{dN}{d\tau} \propto \alpha \left( \frac{h\nu}{m_e c^2} \right)^2 \propto (1 + z)^2 \]

The acceleration is impulsive like in free-free emission so spectral evolution is

\[ \frac{\partial n}{\partial y} = A \frac{1 - e^{-x}}{x^3} \left( \frac{1}{e^x - 1} - n \right) \]
where \( A \) is the ratio of photon creation via double photon Compton scattering or free-free emission to the increase of \( y \). For double photon Compton scattering \( A \) scales like \((1 + z)\). Longest lasting distortion has \( n = (\exp(x + \mu(x)) - 1)^{-1} \) with \( \mu(x) = \mu_o \exp(-x_o/x) \) and \( x_o = \sqrt{A} \).

\[
\frac{\partial N}{\partial y} = \int x^2 \frac{\partial n}{\partial y} dx
\]

\[
= \int x^2 A \frac{1 - e^{-x}}{x^3} \left( \frac{1}{ex - 1} - n \right) dx
\]

\[
= A \frac{\mu}{x_o} = \mu \sqrt{A}
\]

Since the deficit of photons associated with \( \mu \) is \( \mu \pi^2 / 3 \), one finds a thermalization rate per unit \( y \) of

\[
\frac{\partial \ln \mu}{\partial y} = \frac{3 \sqrt{A}}{\pi^2} \propto \sqrt{1 + z}
\]  

(1)
2.5 months after Big Bang

Since \((1 + z)\partial y/\partial z \propto \Omega_B h^2(1 + z)^2\), the overall rate for eliminating a \(\mu\) distortion scales like \(\Omega_B h^2(1 + z)^{5/2}\) per Hubble time. A proper consideration (Burigana et al. 1991, ApJ, 379, 1-5) of this interaction of the photon creation process with the Kompaneets equation shows that the redshift from which \(1/e\) of an initial distortion can survive is

\[
z_{th} = \frac{4.24 \times 10^5}{[\Omega_B h^2]^{0.4}}
\] (2)

which is \(z_{th} = 1.9 \times 10^6\) for \(\Omega_B h^2 = 0.0224\).
The big change at $z = 10^9$ is when the electron-positron pair plasma annihilated. Any $dU/U$ at $z < 2 \times 10^6$ is limited to $dU/U < 6 \times 10^{-5}$.
CONCLUSIONS

• FIRAS did so well because it only measured the difference between the spectrum and a blackbody.

• SYMMETRY and USING the CORRECT FREQUENCY count for more than raw sensitivity.

• Less that 60 parts per million of the CMB energy has been added by warm electrons since $z=2\times10^6$ only 2-3 months after the Big Bang.

• Cosmic far-IR background is 4% of CMB so it must have a spectrum quite similar to the Milky Way spectrum to avoid FIRAS limits.