Characteristic Curve (H&D) Calibration of the Mt. Wilson Solar Photographic Archive Ca K Spectroheliograph Image Sequence

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This note describes our approach to utilizing the stepwedge exposures on the CaK Spectroheliograph sequence. These exposures begin October 9, 1961 and were continued until the end of the sequence. The optical format and photographic plate material both change on October 8, 1962 in that the solar image size increased from 2 inches diameter to 3.4 inches. The glass plate thickness also decreased as part of the change.

Our approach to interpreting the transparency of the photographic plate material in terms of an exposure quantity follows ideas presented by de Vaucouleurs[1]. Our images including both the solar spectroheliograms and a series of stepwedge exposures are returned from an Eskographics Scanmate F14 operated in a 16-bit greyscale mode as tiff files. The original pixels have about 12-bits of significant greyscale information which is distributed over a range of 0 to $2^{16}-1$. The distribution of pixel values in the original data is in the form of comb with about 1 in 8 pixels being populated. The images provided in the fits files have been spatially truncated to record only the average over a 3×3 array. These numbers then have close to 16 bits of significance.

Regular scanner calibration scans set the returned data numbers so that points with no obstruction returns 32767 and points with total obstruction return 0 (we start with signed 16-bit integers and use only the positive values). We define these data numbers to be P. If the photographic plate were a linear detector without fog, the plate darkening would be proportional to the flux of radiation incident during the exposure. In this case we would be able to identify $y = 32767 - P$ as an exposure parameter E where $E = I\delta t$ with I being an incident flux and δt being an exposure time. The fits format files available from the web site directory `MW_SPADP/CaK_fits` provide images with y as the array parameter. In fact, the scanner measures the flux transmitted through the exposed plate. It is common following Hurter and Driffield[2] to convert the data numbers to a photographic density D based on the obscuration of the scan beam by the exposed plate. The ratio of the data number P at a point on the image to the data number P_{clear} with the photographic plate out of the beam is the transmittance and its inverse is called the opacity $O = P_{\rm clear}/P$. Our scanner adjusts the digitization values so that the maximum 16-bit data number is returned for the clear condition. Since we use signed integers and include only the positive case we have $P_{\rm clear} = 32767$. The photographic density D is then taken to be $D = \log_{10}(O)$. Typically a photographic material after development will reach a maximum density $D_{\rm max}$ when fully exposed. A plot between $\log_{10} E$ as the abscissa and D as the ordinate is called the characteristic curve. Typically there is a range of E over which $D \propto \gamma \log_{10} E$ with γ being a function of the emulsion and the development process.

The use of the characteristic curve to interpret photographic material has frequently been limited to those portions of the exposure where the charactistic curve is linear. The usual representation of the characteristic curve is to find a quantity called the inertia which is the exposure offset E_0 where the plate fog intersects the extension of the straight line portion of the characteristic curve. However, the discussion by de Vaucouleurs[1] shows the advantage of taking the zero point on the ordinate instead of the abscissa so that we start with a representation:

$$E = (O - O_{\rm fog})^{1/\gamma}$$

where $O_{\rm fog}$ is the opacity of the plate fog. For the purpose of describing the properties of an image qualitatively, the γ parameter is the primary indicator of image contrast and the plate fog parameter replaces the plate inertia. The above approach produces an extended range of exposure over which there is an approxiately linear relationship between $\log(E)$ and $\log(O - O_{\rm fog})$. As the exposure nears saturation, the slope of the characteristic curve decreases until it either becomes flat so that density is no longer a function of exposure. In extreme cases the curve turns over and enters a state called solarization where the density is decreased by further exposure. For the Mt. Wilson photographic material, we have not encountered any cases of solarization so we will ignore this condition. We will, however, address the question of plate saturation as that is a common condition in the collection.

We use a formulation of the characteristic curve fitting problem which continues from the idea of de Vaucouleurs[1] wherein we alter the photographic density axis by subtracting an offset corresponding to the plate fog and use a quantity called the opacitance by De Vaucouleurs which is:

$$\omega = O - 1 \quad \text{or} \quad \omega = \frac{32767 - P}{P} = \frac{y}{32767 - y} \ .$$

We use a fit of the form:

$$\omega - \omega_{\rm fog} = aE^{\gamma} \ .$$

We use known relative values of E from the step wedge exposures:

$$E_i = bW_i$$

with the values of W_i being taken from the Leighton logbook entry shown in figure 1 to find values of γ and a from a least square fit and we have taken $\omega_{\rm fog}$ to be the minimum

Step No.	Dist. from Right End (mm)	Trans-mission (Red)	Trans-mission (Violet)	Step No.	Dist. from Right End (mm)	Trans-mission (Red)	Trans-mission (Violet)
0	1.5	1.00	1.00	8	41.5	.100	.11
1	6.5	.97	.98	9	46.5	.055	.055
2	11.5	.89	.89	10	51.5	.028	.028
3	16.5	.77	.77	11	56.5	.013	.012
4	21.5	.60	.61	12	61.5	.0061	.0053
5	26.5	.45	.47	13	66.5	.0028	.0021
6	31.5	.30	.32	14	71.5	.0014	.0010
7	36.5	.18	.19	15	77.5	.0008	.0005

Step wedge measured 6-30-61 (See notebook)

Figure 1: This image shows the log book page where Robert Leighton recorded the transmission factors for the step wedges used for the conversion of plate transmission factors into plate exposures. We take the values of W_i defined in the text from the columns labeled "Transmission (Violet)".

of ω_i from the identified step wedges. The value of b is an arbitrary scale factor that we choose for convenience.

The step wedge rectangles have been imprinted by exposing the plate to a controlled lightbox after the solar image is obtained. The location of the step wedge images is fixed by holding the plate against an angle bracket in the dark room. Due to variations in the implementation of this exposure, the rectangle nearest the plate edge is not always identifiable so that the automatic identification of the step wedge strips does not always find the rectangle corresponding to $i = 0$ in the above table and we define i_1 as the first identifiable rectangle. In order to preserve the same numerical range as the original image, we pick b as $b = \omega_{i_1}/W_{i_1}$ so that $E_{i_1} = \omega_{i_1}$. For $i = i_1$ the wedge has the greatest transparency and the rectangle on the plate is most exposed. This scale factor will be replaced subsequently as part of a fit to a center-to-limb intensity function for the solar

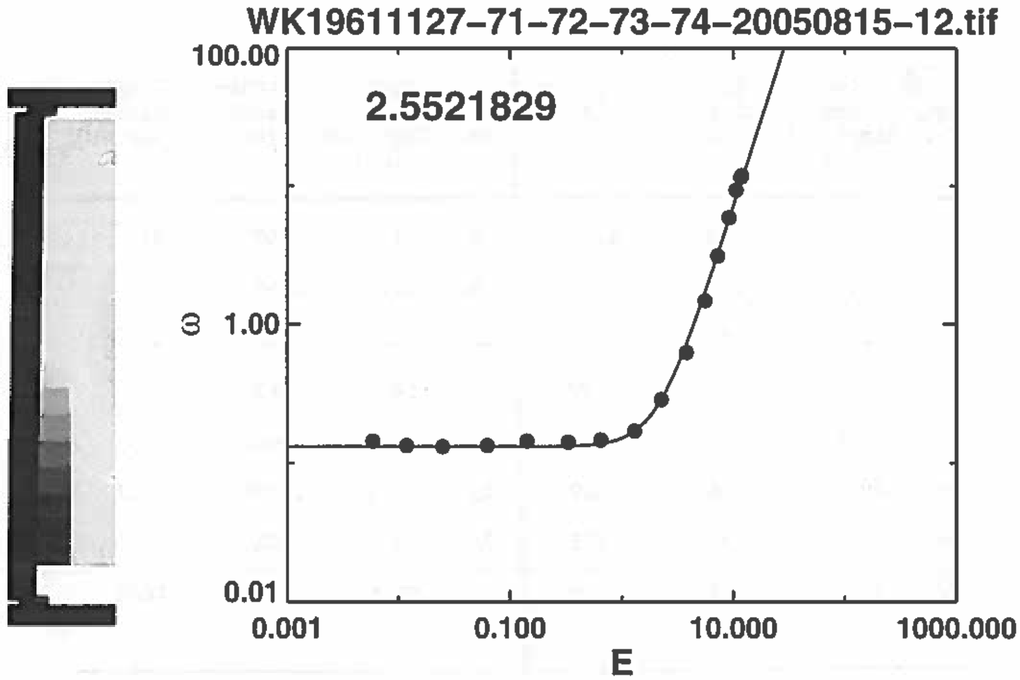

Figure 2: This figure shows an example of a step wedge image on the left and the derived fit to the opacitance on the right. In this case, the most heavily exposed step wedge rectangle was correctly identified and the highest opacitance corresponds to the darkest part of the image on the bottom. This postscript image only includes eight-bit significance whereas the actual analysis has been done on the full significance image. The fit is labeled with the name of the extracted stepwedge file at the top and the determined value of γ inside the upper left corner of the plot.

image. The coefficients a and $\omega_{\rm fog}$ are then found using the values of ω_i corresponding to each step wedge portion of the exposed plate and fitting the coefficients using the scaled values of $E_i = \omega_{i_1}(W_i/W_{i_1})$.

The inverse formula needed to calculate a linear image with variable E in place of the observed data numbers ω is then:

$$E = \left(\frac{\omega - \omega_{\rm fog}}{a}\right)^{1/\gamma} .$$

Figure 2 shows both a sample step wedge image and the fit of its opacitance to the fitting formula.

A modification we make concerns the treatment of the data points near the fog limit. For these, both positive and negative signs of $\omega - \omega_{\rm fog}$ must occur since we have selected $\omega_{\rm fog}$ to be an average of ω in these clear plate regions. For a case where $\gamma = 1$, there is no problem with this result. However, when γ differs from 1, we will be unable to take roots of the negative numbers. The points with $\omega < \omega_{\text{fog}}$ could be thrown out but that would bias averages near the fog limit. Instead we the Fortran sign function to write:

$$E = \textbf{sign}\left(|\omega - \omega_{\text{fog}}|^{1/\gamma}, \ \omega - \omega_{\text{fog}}\right)$$

where

$$\textbf{sign}(u, v) = |u| \ \text{ for } v >= 0 \quad \text{and} \quad \textbf{sign}(u, v) = -|u| \ \text{ for } v < 0 \ .$$

This approach alters the statistical distribution of the linearized points near $\omega = \omega_{\text{fog}}$ but preserves the fact that ω averages to ω_{fog} in regions of the plate where there is only plate fog by yielding an average of E that is zero.

References

1. de Vaucouleurs, G.: 1968. *Appl. Opt.* **7**, 1513–+.

2. Hurter, F. and F. V. Driffield: 1890. *Jour. Soc. Chem. Ind.* **9**, 455–469.