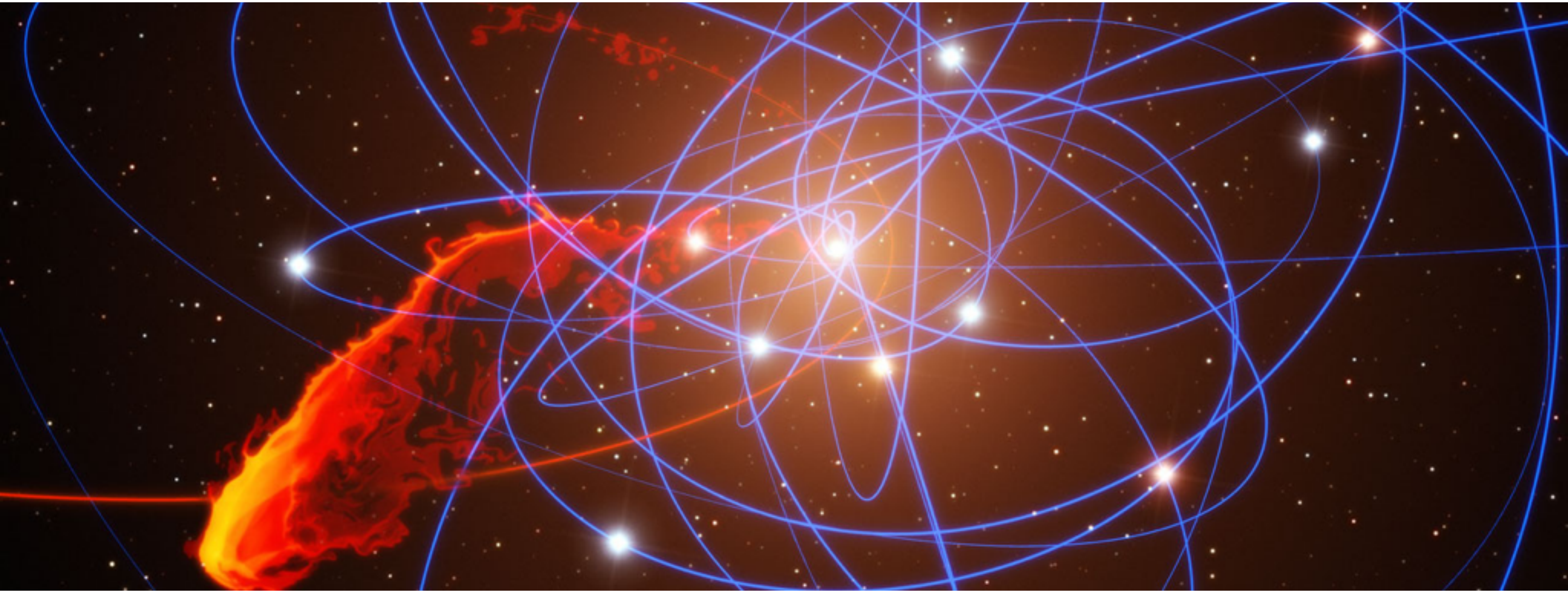

On constraining the spin of the MBH the GC via star orbits: the effects of stellar perturbations

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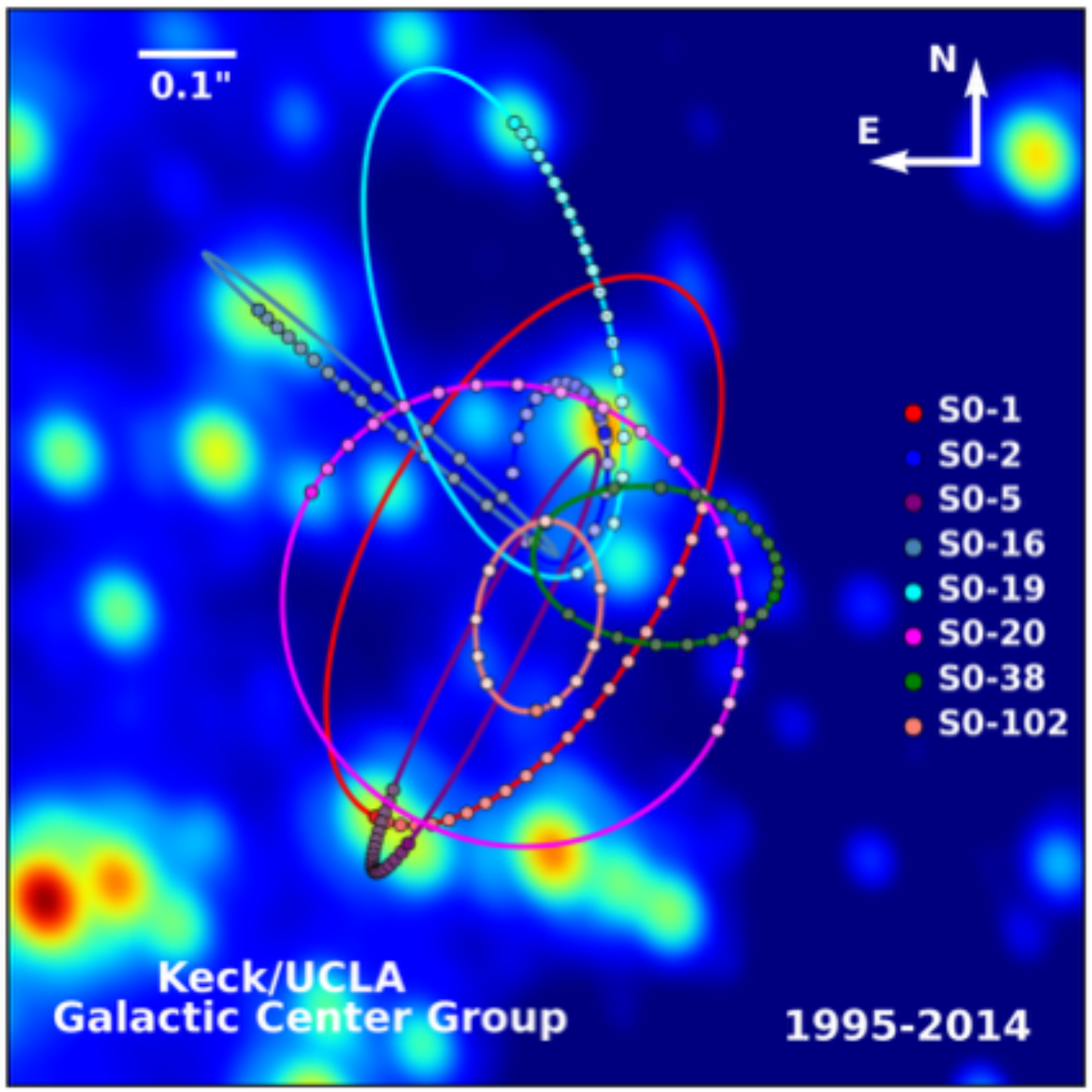
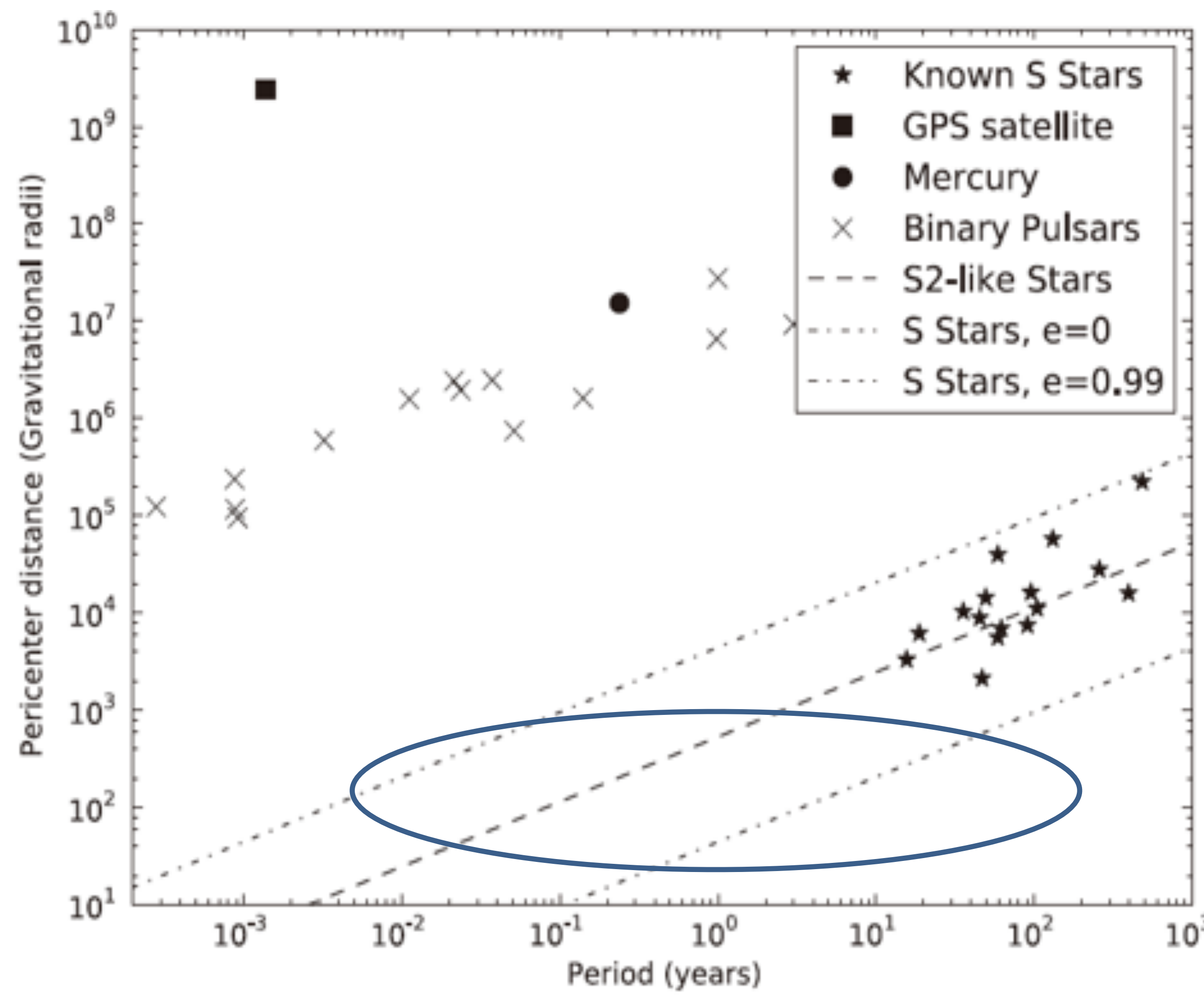
2016. 2. 11, Aspen Center for Physics

Strong field GR test and the GC S-stars

- **Clusters of young stars in the GC**
- Provide a unique environment of testing GC by stellar orbits



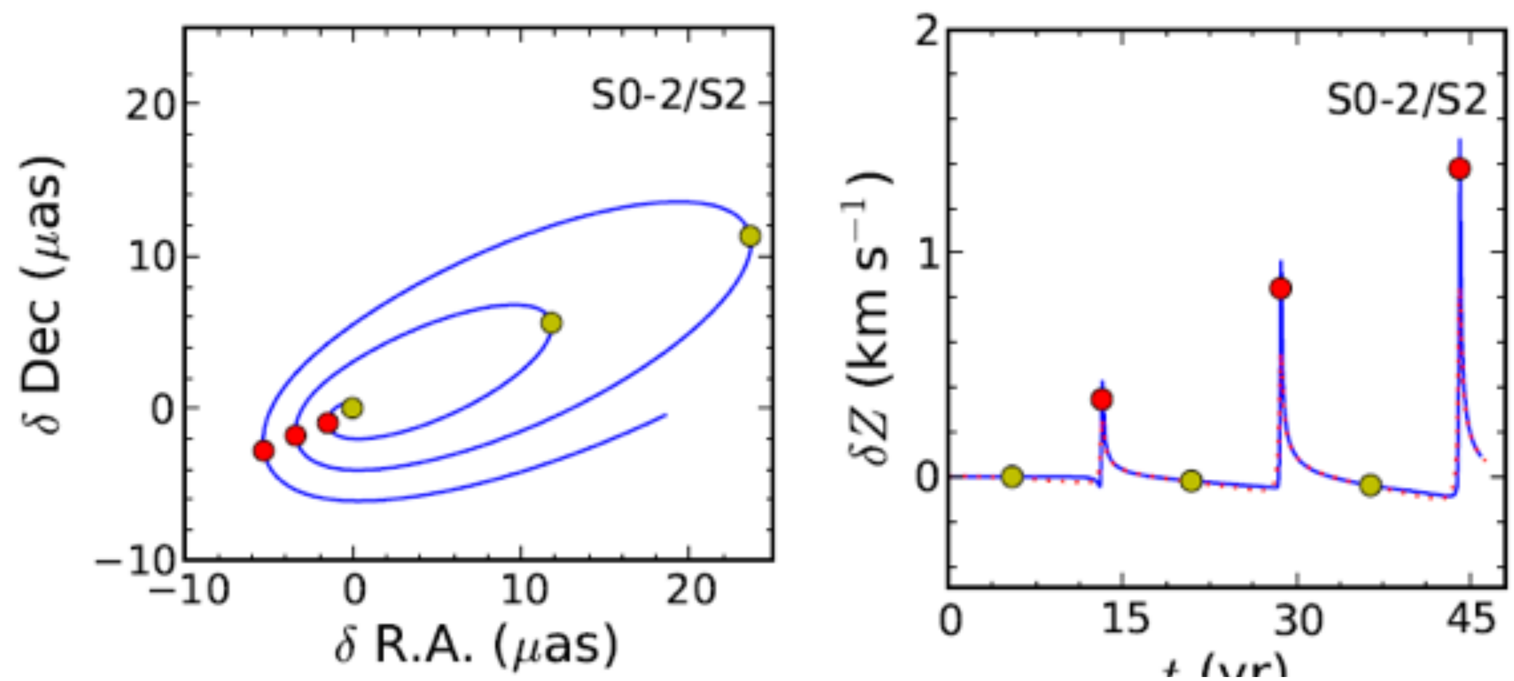


http://www.galacticcenter.astro.ucla.edu/

Angelil et al. 2010

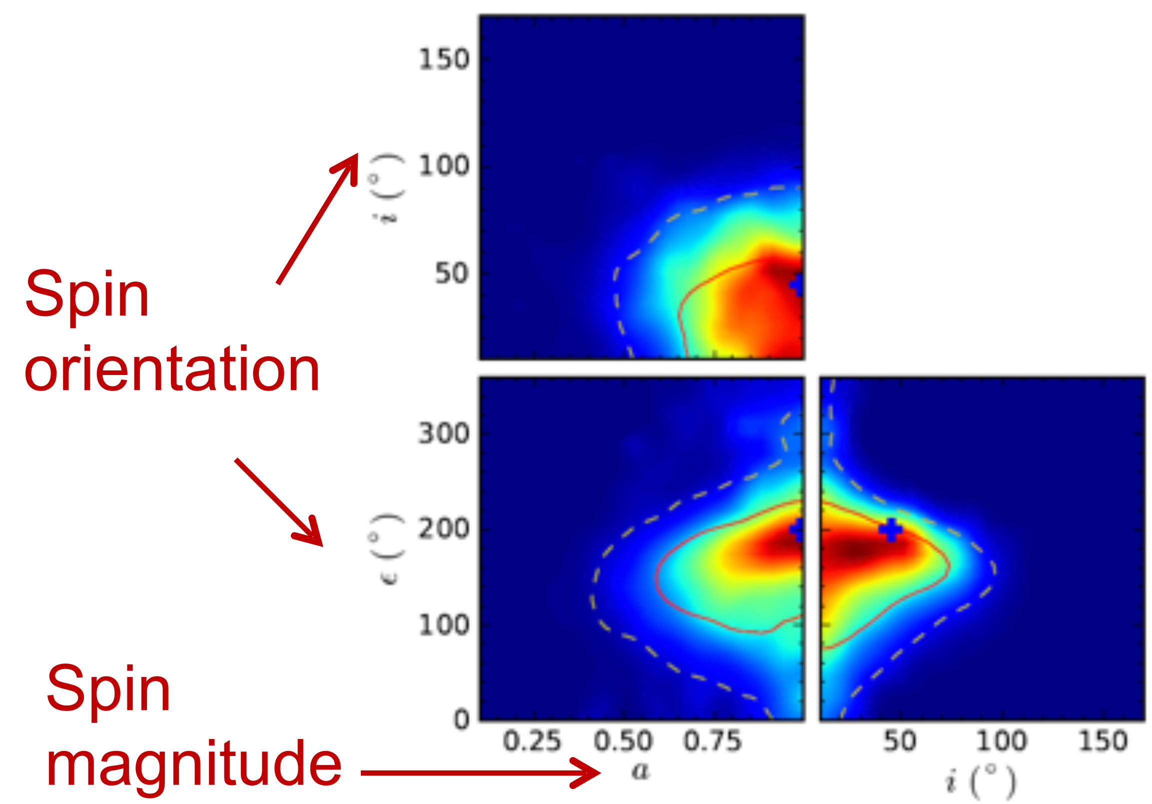
Our previous works

- **The constraints of the spin parameters by observing the trajectories and the redshifts of the S-stars by future facilities** (Zhang, Lu, & Yu 2015; Yu, Zhang, & Lu, submitted)



  - **Full GR treatment**
  - **MCMC fitting**
    - Magnitude and direction of spin, 6 orbital elements, MBH and R_{GC}

- **We can constraint the spin by observing the orbits of S2 or other inner S-stars**



- **But stellar perturbations are not considered**

Perturbations

- **Stars**： Early and late type stars（Bartko, et, al. 2010）
- **Stellar remnants**
  - Stellar mass black holes: Mass segregation（Freitag, et, al. 2006）
  - Neutron stars, pulsars, white dwarfs: (Morris 1993)
- **Intermediate mass black hole(s)**： 100-1000 solar mass, distance>200 AU (Yu & Tremaine 2003, Gualandris & Merritt 2009; etc)
- **Dark matter**

- Distinguish
  - Gravitational perturbations from background sources
  - Spin-induced perturbations
  - Different predictions from other gravity theories（e.g., f(R) theory)

Previous studies

- **Post-NW approximation** (Merritt et al. 2010)
  - Frame-dragging obscured beyond 0.5mpc

$$\mathbf{a}_{J,1} = -\frac{3G^2M_\bullet}{c^3}\sum_{j\neq 1}\frac{m_j}{r_{1j}^3}\left\{\left[\mathbf{v}_{1j} - (\mathbf{n}_{1j}\cdot\mathbf{v}_{1j})\,\mathbf{n}_{1j}\right]\times\boldsymbol{\chi} - 2\mathbf{n}_{1j}\,(\mathbf{n}_{1j}\times\mathbf{v}_{1j})\cdot\boldsymbol{\chi}\right\},$$

$$\mathbf{a}_{J,j} = \frac{2G^2M_\bullet^2}{c^3 r_{1j}^3}\left\{\left[2\mathbf{v}_{1j} - 3\,(\mathbf{n}_{1j}\cdot\mathbf{v}_{1j})\,\mathbf{n}_{1j}\right]\times\boldsymbol{\chi} - 3\mathbf{n}_{1j}\,(\mathbf{n}_{1j}\times\mathbf{v}_{1j})\cdot\boldsymbol{\chi}\right\},$$

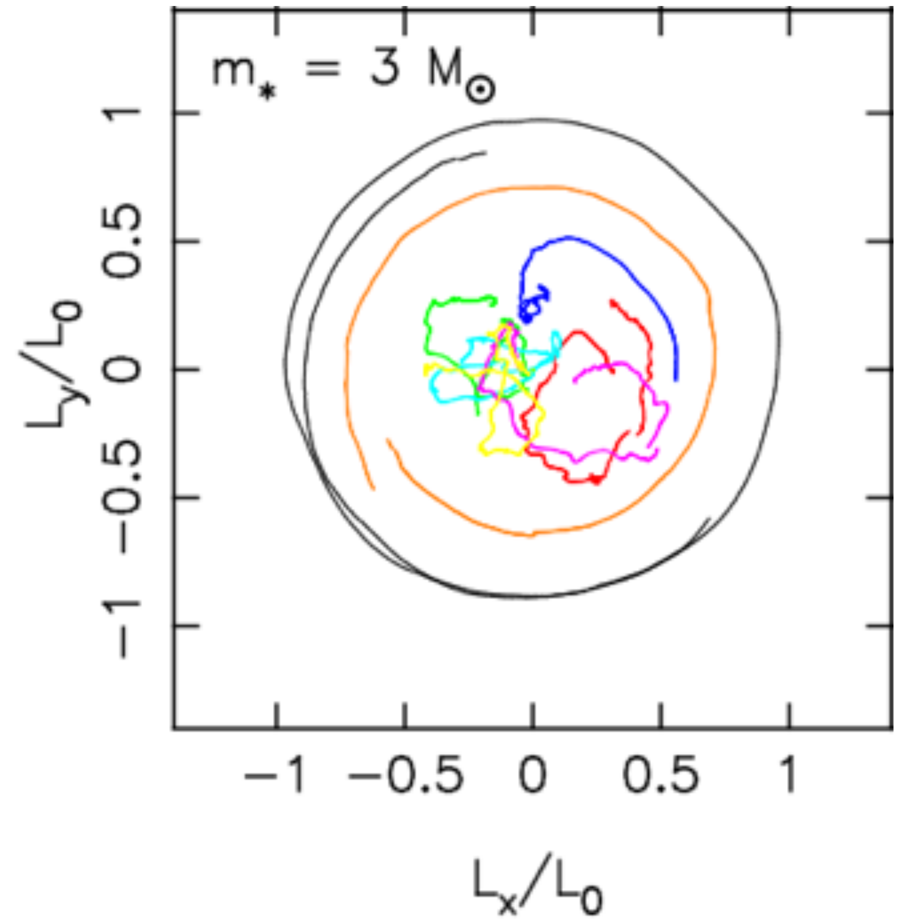
$$\dot{\boldsymbol{\chi}} = \frac{G}{2c^2}\sum_{j\neq i}\frac{m_j}{r_{ij}^2}\left[\mathbf{n}_{1j}\times(3\mathbf{v}_1 - 4\mathbf{v}_j)\right]\times\boldsymbol{\chi},$$

$$r_{ij} \equiv |\mathbf{x}_i - \mathbf{x}_j|, \quad \mathbf{x}_{ij} \equiv \mathbf{x}_i - \mathbf{x}_j, \quad \mathbf{n}_{ij} = \mathbf{x}_{ij}/r_{ij}, \quad \mathbf{v}_{ij} \equiv \mathbf{v}_i - \mathbf{v}_j.$$

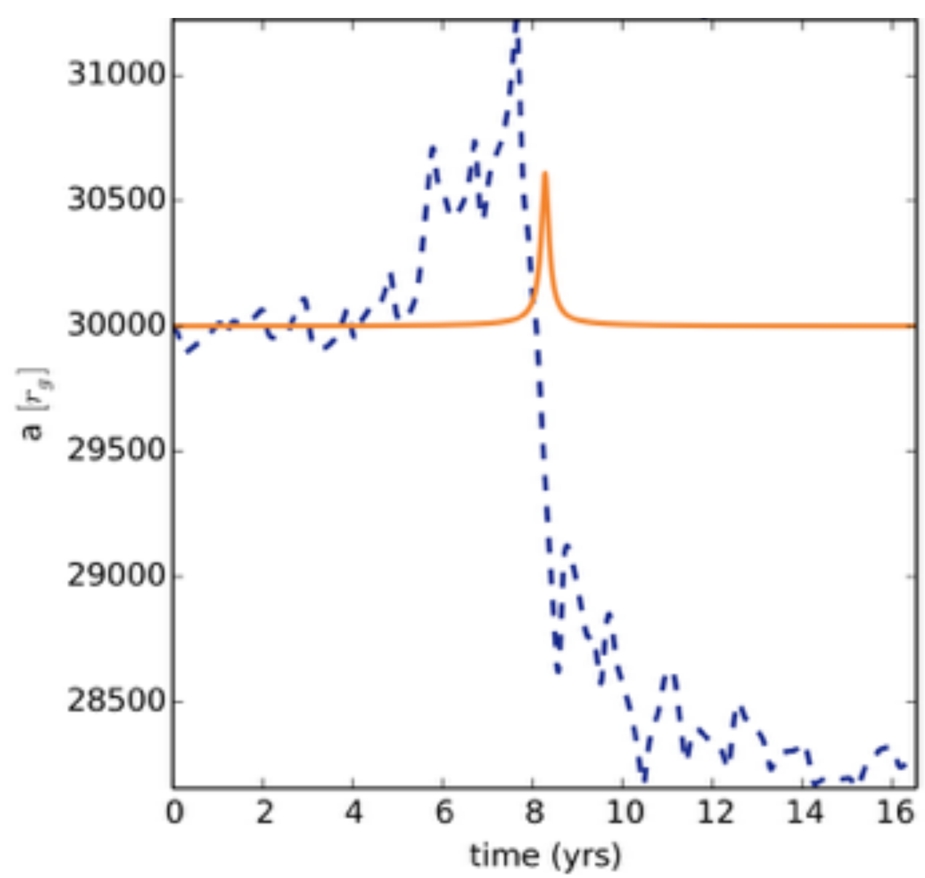
- **Hamiltonian perturbation** (Angelil & Saha 2014)
  - Frame-Wavelet decomposition

$$H_{\mathrm{stellar}} = \sum_j \frac{m_j}{M}\left(\frac{\boldsymbol{x}\cdot\boldsymbol{x}_j}{|\boldsymbol{x}_j|^3} - \frac{1}{|\boldsymbol{x}-\boldsymbol{x}_j|}\right),$$

- **Orbital perturbation theories**
  Sadeghian & Will 2011; Iorio 2011; etc



Merritt et al. 2011

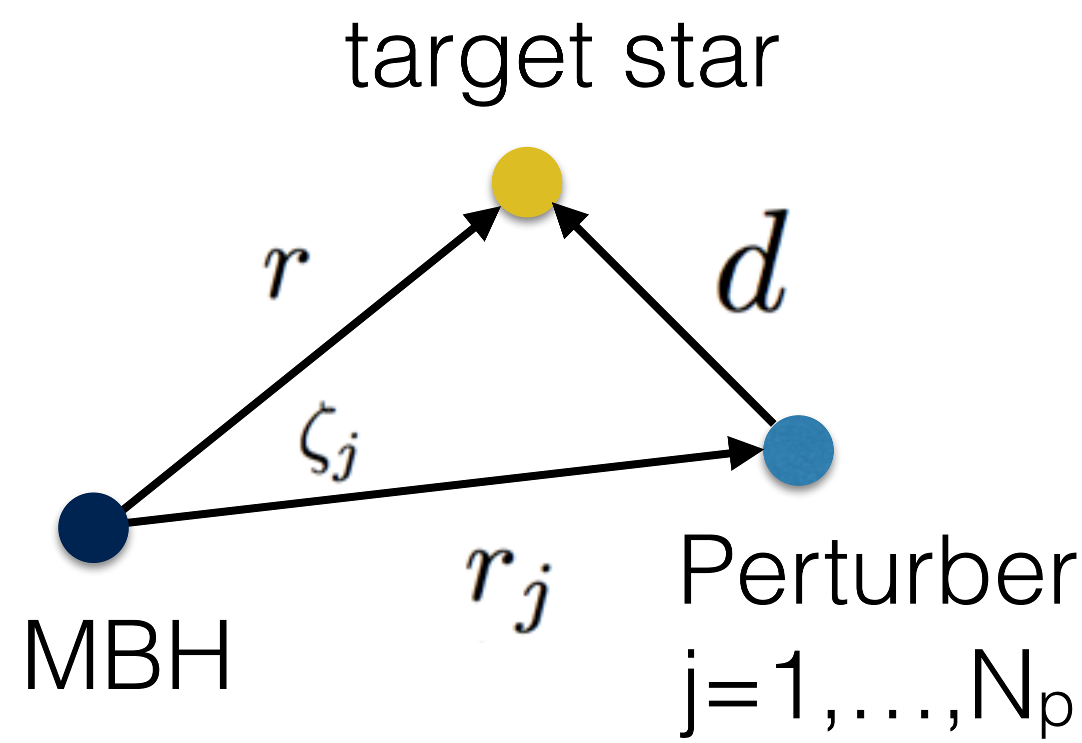


Angelil & Saha 2014

Motion of the perturbed target star

- **Hamiltonian contributed by perturbation** (Angelil & Saha 2014, Wisdom & Holman 1991)

$$H_{\rm p} = \sum_{j}^{N_p} m_{{\rm p},j} \left(\frac{r}{r_j^2} \cos\zeta - \frac{1}{d} \right)$$



- **Simplification**
  - The multure interactions between perturber are ignored
  - The target star is a test particle (mass=0)

- **Motions of the perturbers follows the unperturbed motion equation**

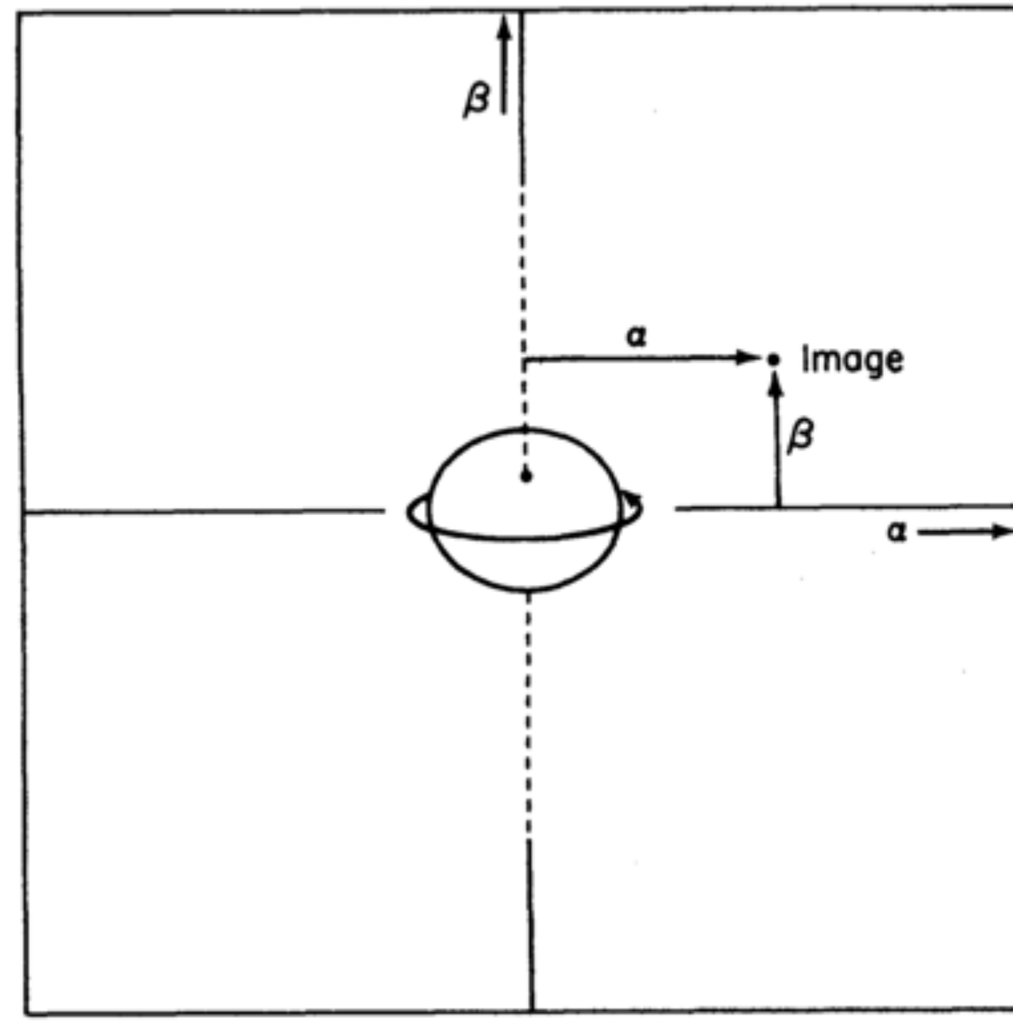
$$\mathbf{H{=}H_0{+}H_p}$$

Light Tracing technique

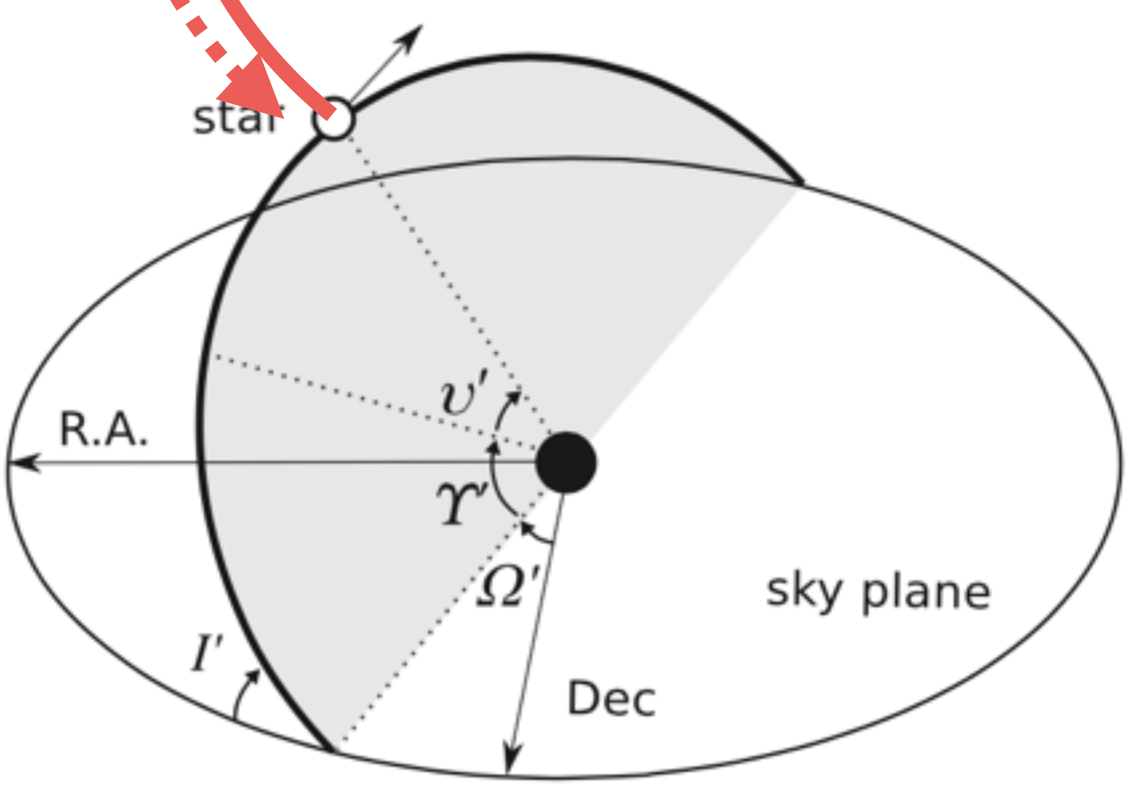
$$\int_{r_0}^{r_{\rm hit}} \frac{dr}{\sqrt{R}} = \pm \int_{\mu_0}^{\mu_\star} \frac{d\mu}{\sqrt{\Theta_\mu}}, \quad \cdots$$

$$\lambda = -\alpha \sin i,$$

$$q^2 = \beta^2 + \left(\alpha^2 - a^2\right)\cos^2 i.$$



Cunningham & Bardeen 1972



$\alpha r_{\rm g}/R_{\rm GC}$

$\beta r_{\rm g}/R_{\rm GC}.$

→

R.A.

Dec

$$Z = \frac{\boldsymbol{p}_{\rm hit} \cdot \boldsymbol{u}_\star}{\boldsymbol{p}_{\rm o} \cdot \boldsymbol{u}_{\rm o}} - 1 = -\frac{\boldsymbol{p}_{\rm hit} \cdot \boldsymbol{u}_\star}{E_{\rm o}} - 1.$$

- **Perturbations on the observational quantities**
  - Positions of the star in the sky at time t

$$\delta\mathrm{R.A.} \qquad \delta\mathrm{Dec} \qquad \delta R(t) = \sqrt{\delta\mathrm{R.A.}^2 + \delta\mathrm{Dec}^2}$$

  - Redshift at time t

$$\delta Z(t)$$

- **Root mean square value (in three orbits)**

$$\delta R_{\rm rms} = \sqrt{\frac{1}{T_{\rm ob}}\int^{T_{\rm ob}} \delta R(t)^2 dt} \qquad \delta Z_{\rm rms} = \sqrt{\frac{1}{T_{\rm ob}}\int_0^{T_{\rm ob}} \delta Z(t)^2 dt},$$

- **Spin-induced effects:**

unperturbed target star, ***a=0.99*** ⟷ unperturbed target star, ***a=0.0***

- **Stellar perturbations:**

perturbed target star, a=0.0 ⟷ ***unperturbed*** target star, a=0.0

- **Total perturbations:**

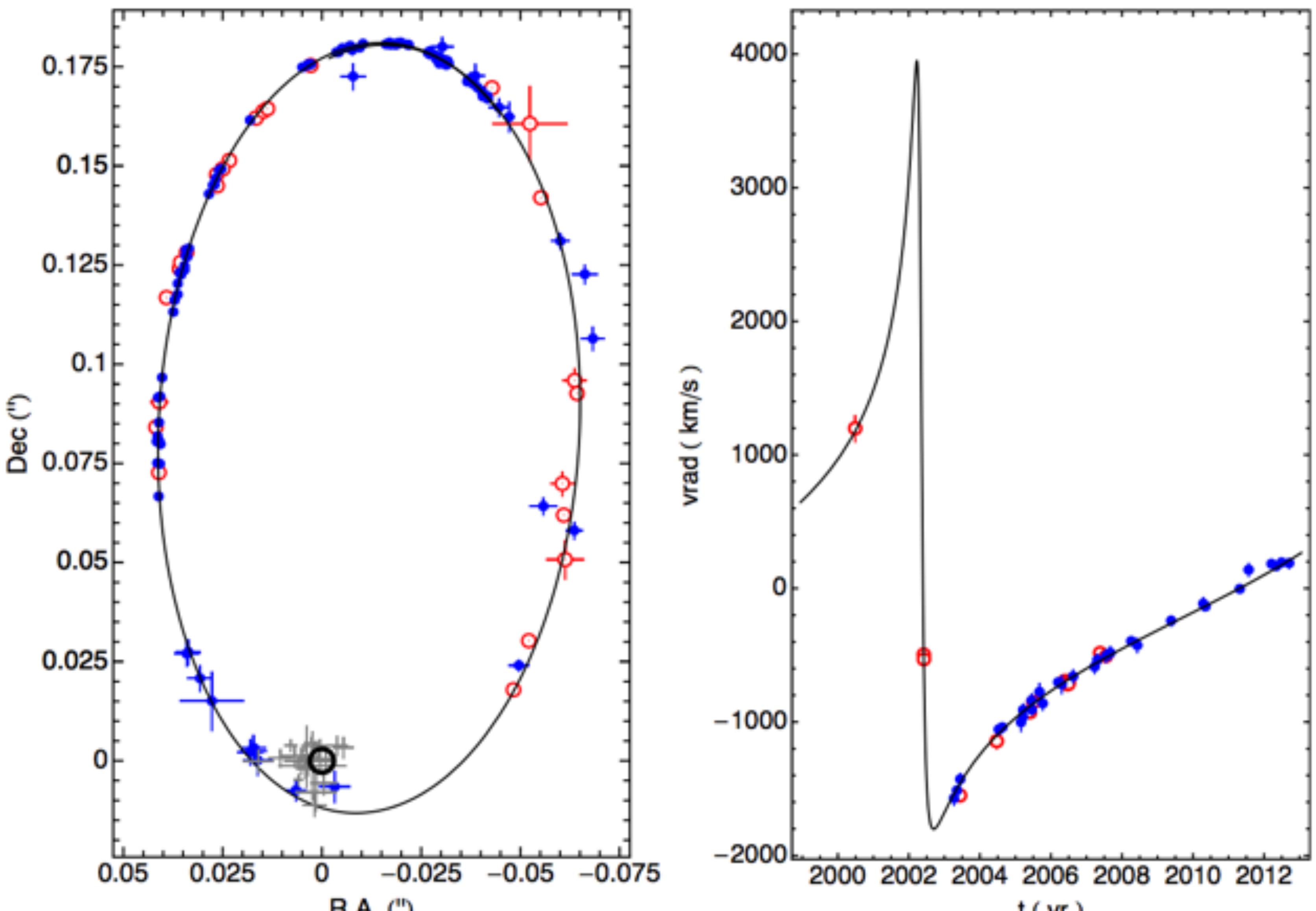
perturbed target star, ***a=0.99*** ⟷ ***unperturbed*** target star, ***a=0.0***

Results: Single Perturber

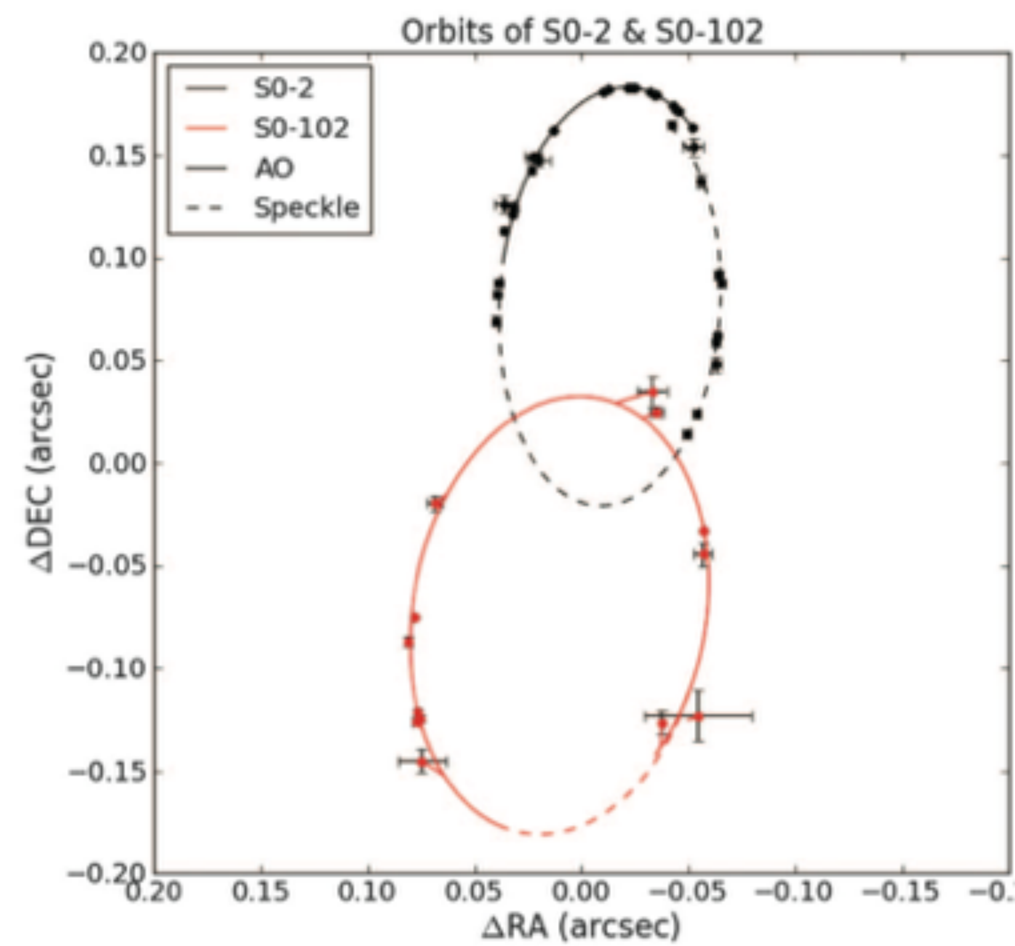
S2/S0-2 and S0-102

- **S2/S0-2**
  - Orbital period of 15 years
  - Pericenter distance of 100AU
- **S0-102**
  - Orbital period of 11 years
  - e~0.68

How S0-102 affects the orbital motion of S2?

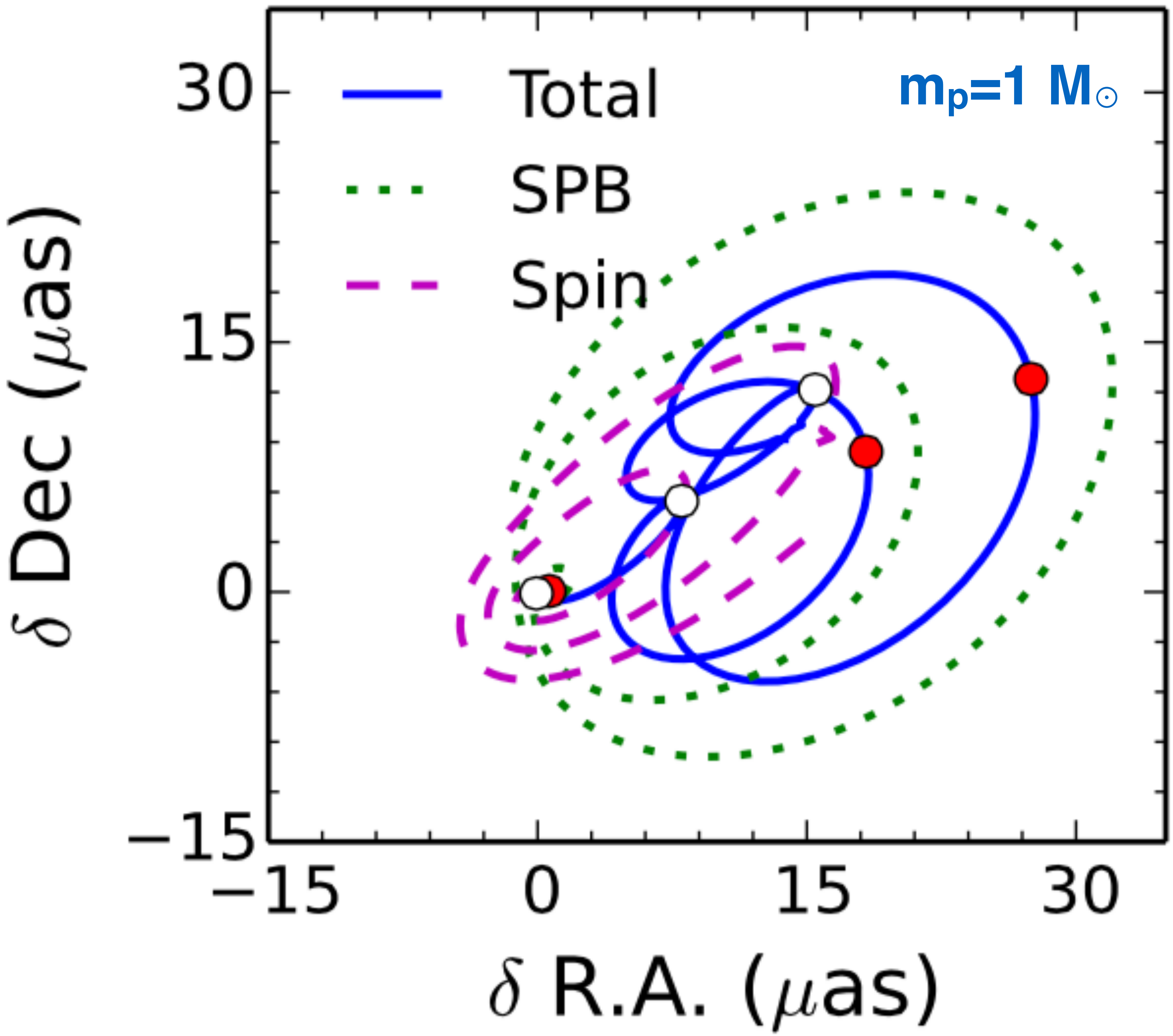


Gillessen et al. 2013

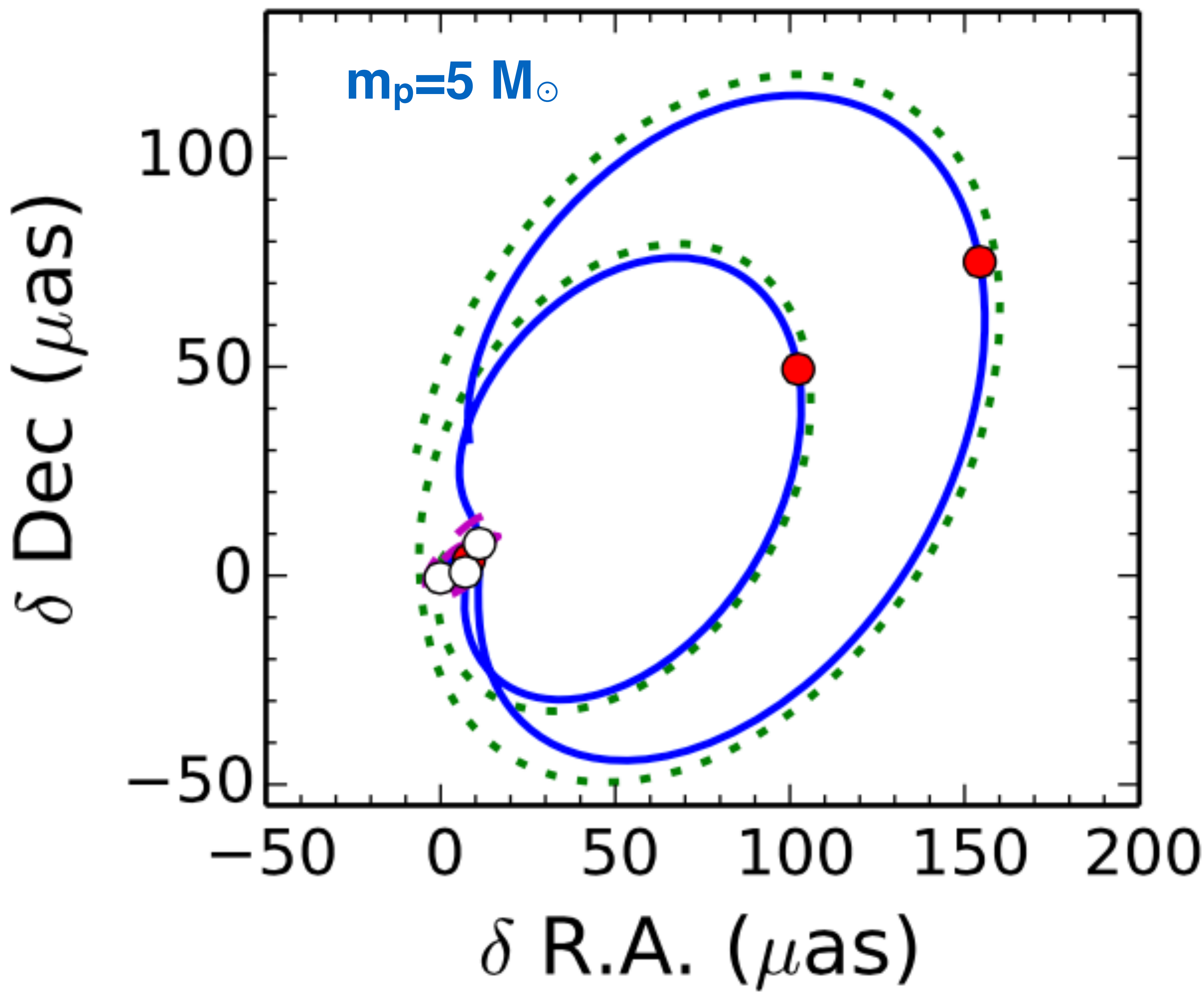


Meyer et al. 2012

S2 perturbed by S0-102



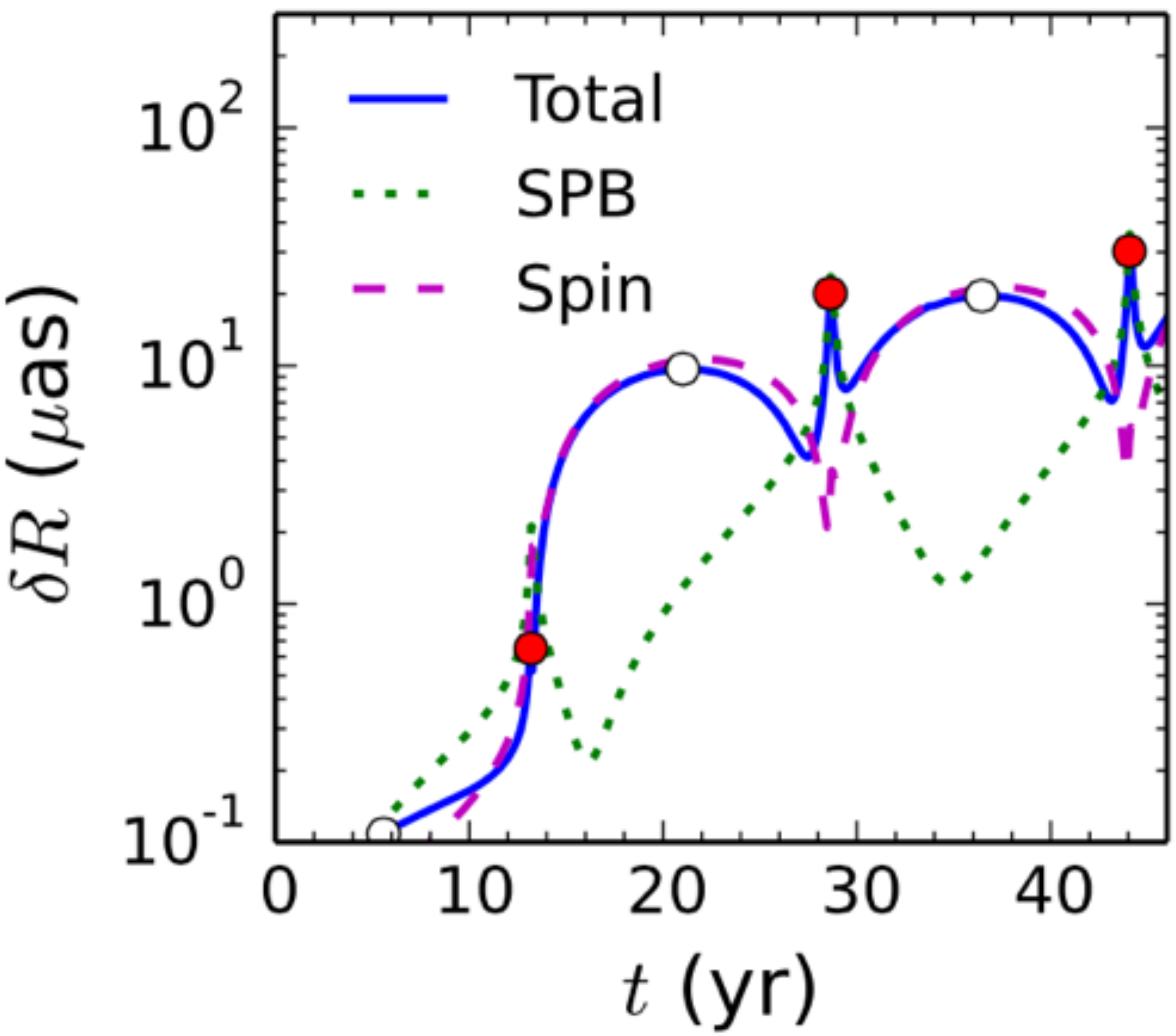
S2 perturbed by S0-102



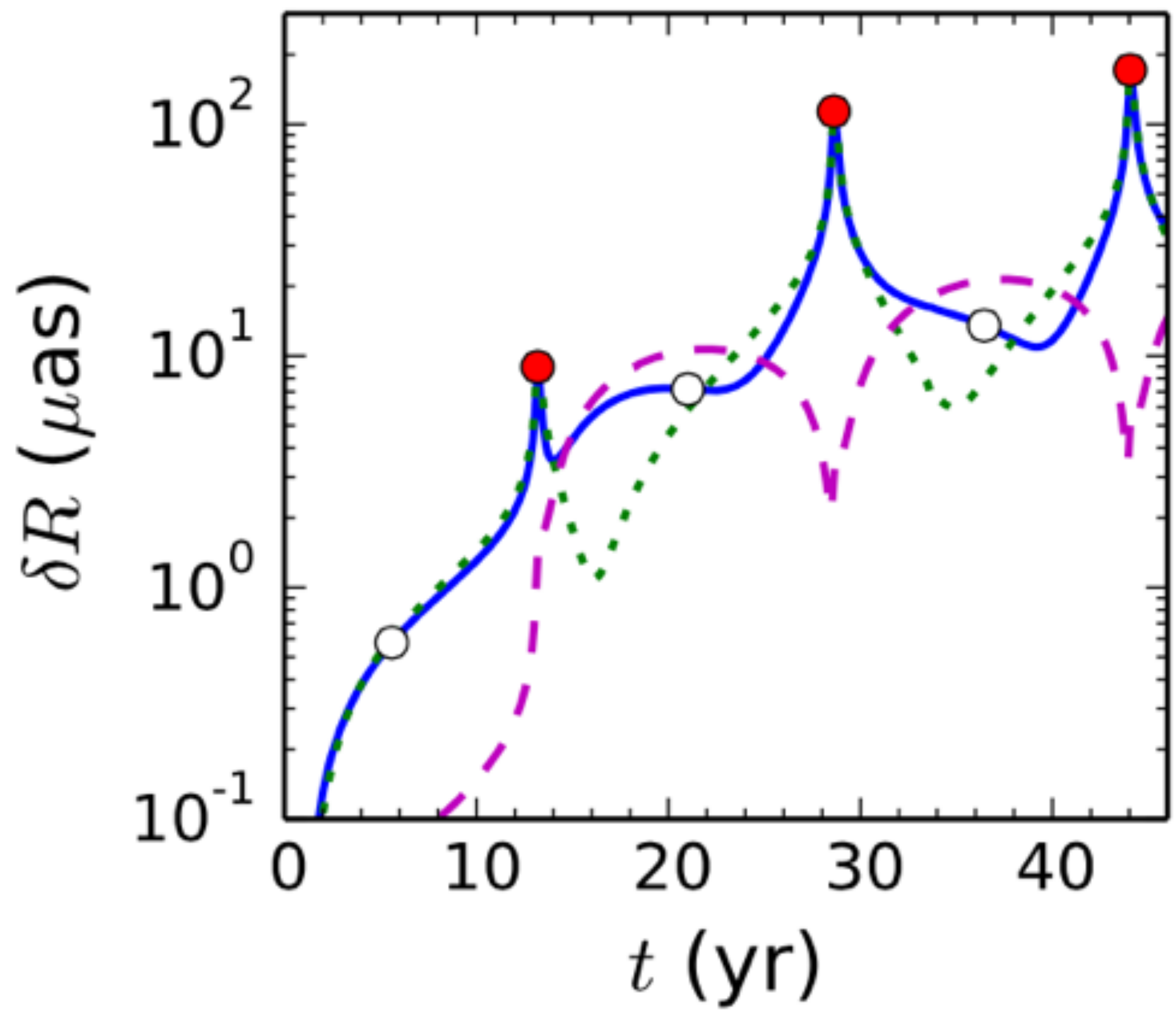
Orbits of S2 perturbed by S0-102

$$\delta R(t) = \sqrt{\delta \mathrm{R.A.}^2 + \delta \mathrm{Dec}^2}$$

- $m_p = 1\ M_\odot$

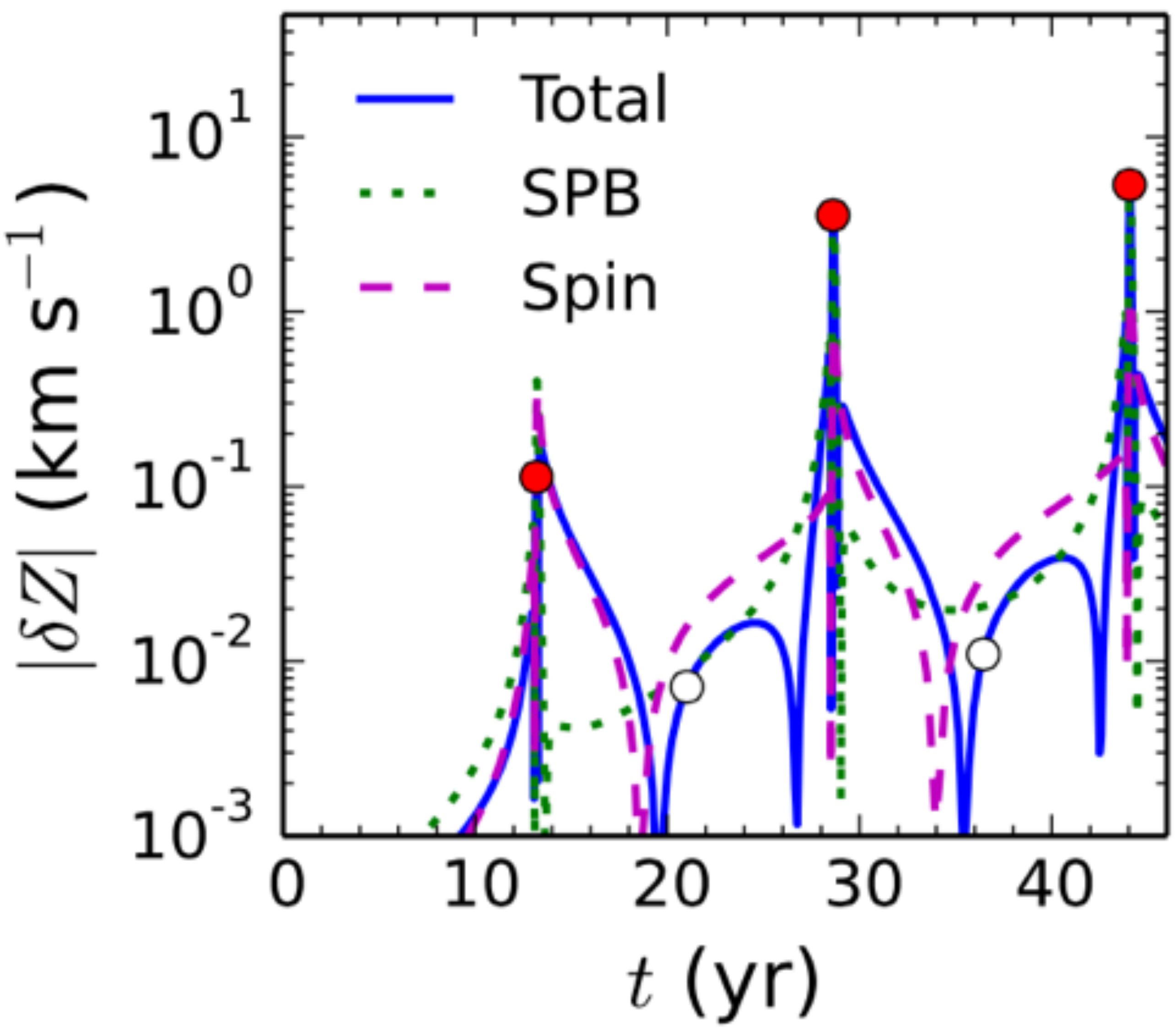


- $m_p = 5\ M_\odot$

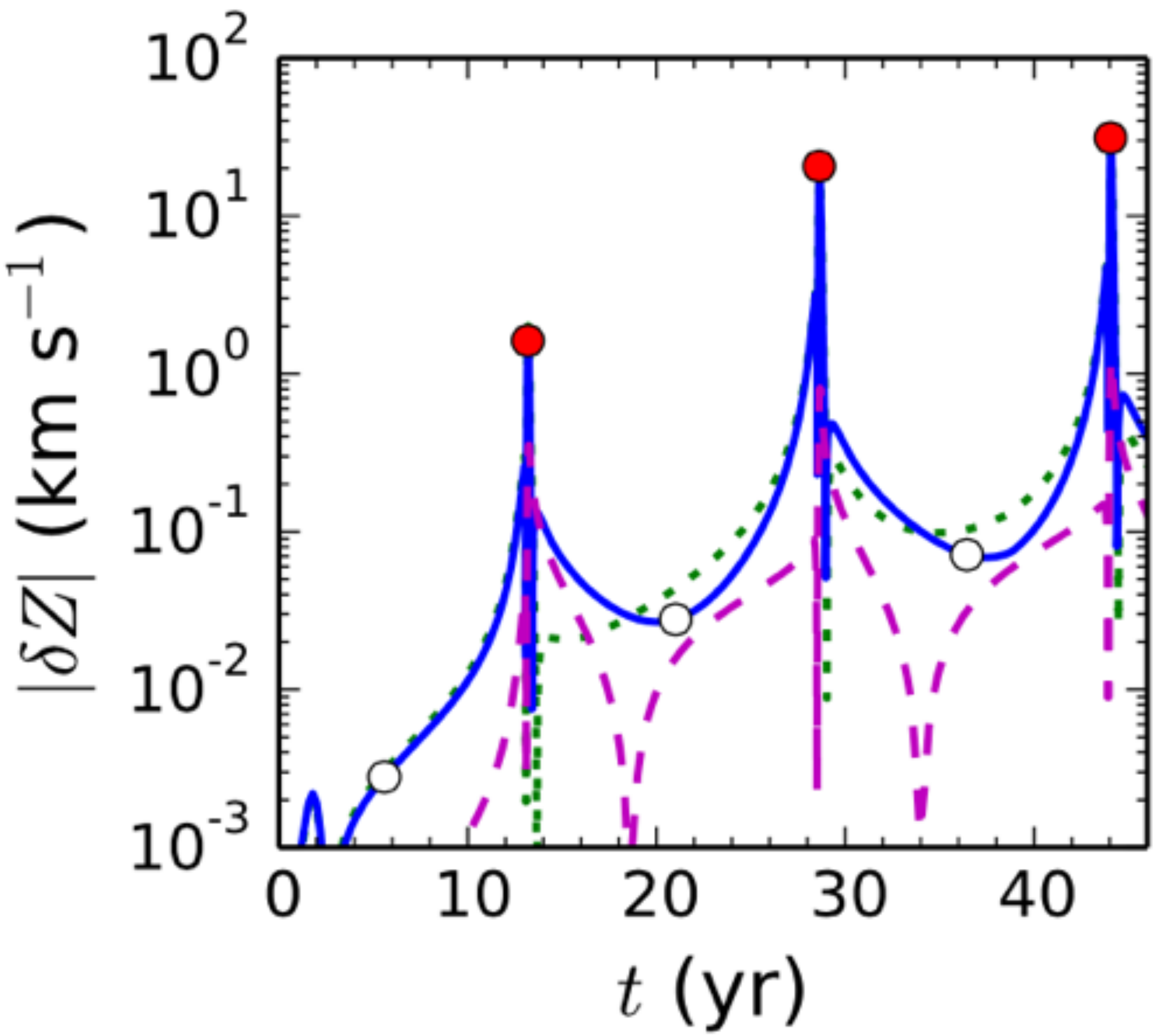


Orbits of S2 perturbed by S0-102

- m_p=1 $M_\odot$

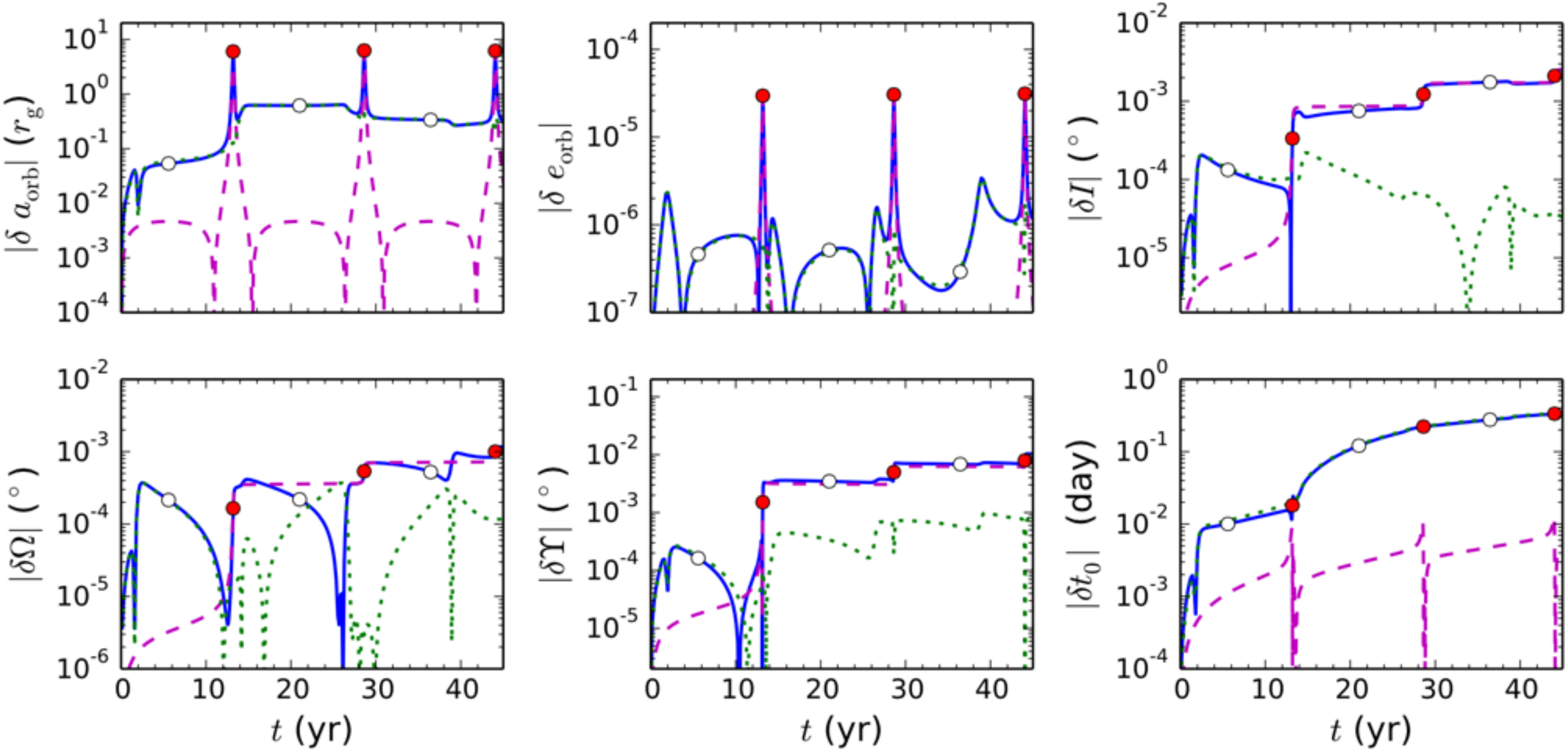


- m_p=5 $M_\odot$



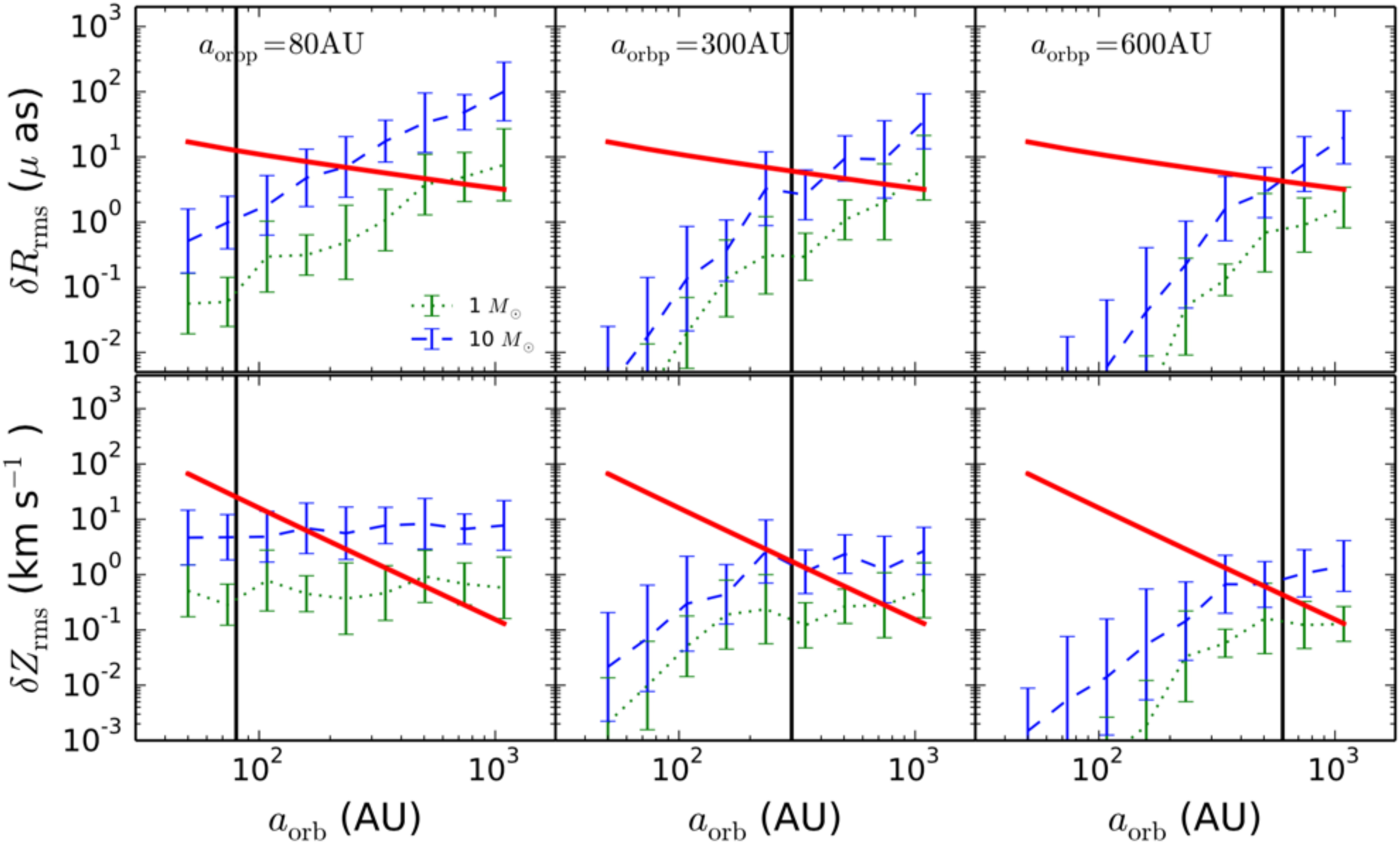
Orbits of S2 perturbed by S0-102

- **Variations of the orbital elements of S2**



- **The orbital period of S2 is perturbed: $|dt_0|$~ 0.3 day after 45 years —> ~40 uas difference in sky position (> spin :10 uas)**

Inner S-stars perturbed by a single perturber



- **The stellar perturbations are dominated by perturbers inside the target star**

Results:
Perturbations due to a star cluster

Star clusters

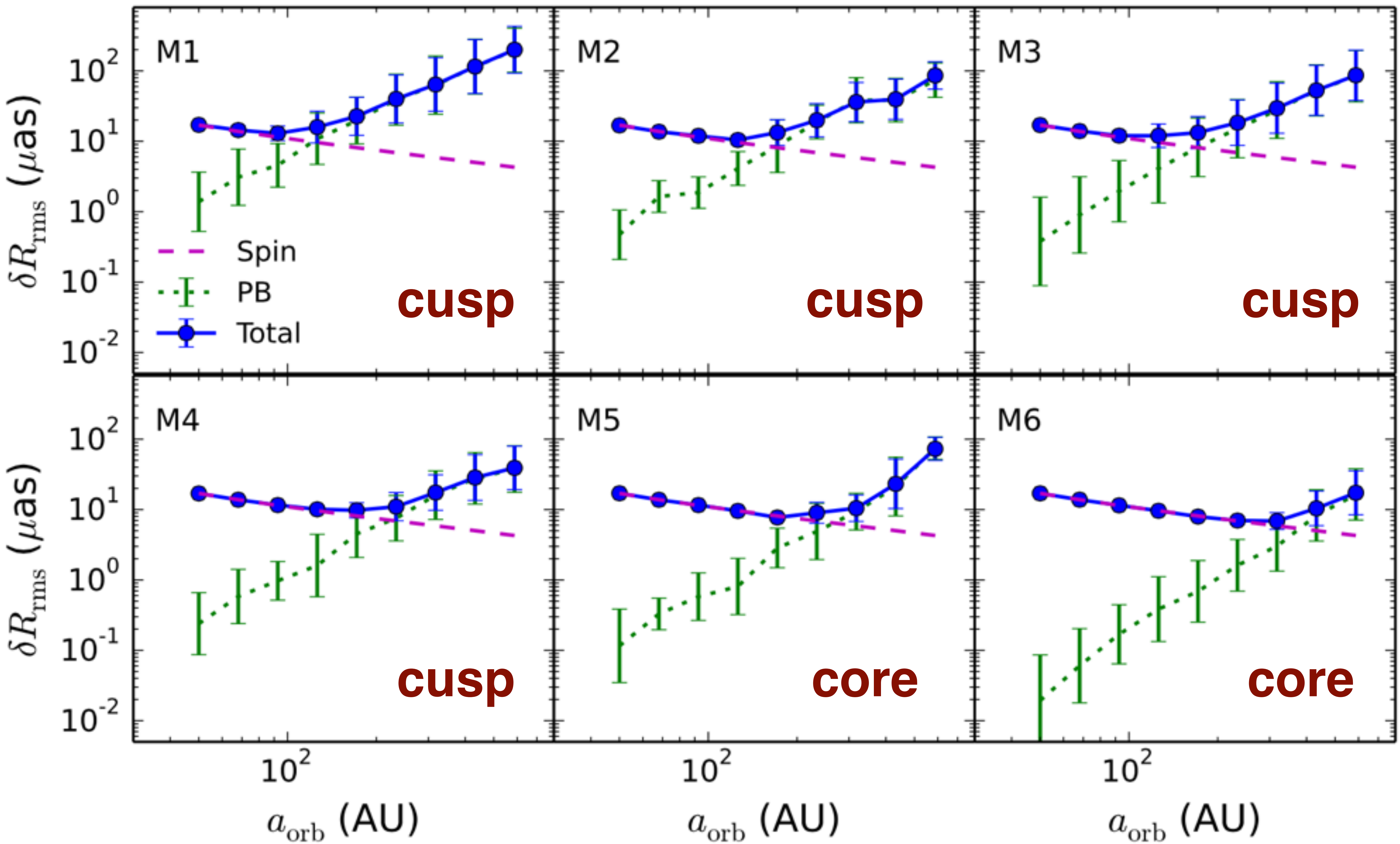
- **Density profile** $n(r) \propto r^{-\gamma}$
  - Bahcall-Wolf Cusp (Bahcall & Wolf 1976) $\gamma = 1.75$
  - Core-like profile (Do et al. 2009) $\gamma = 0.5$
- **Initial conditions (Merritt et al. 2011)**

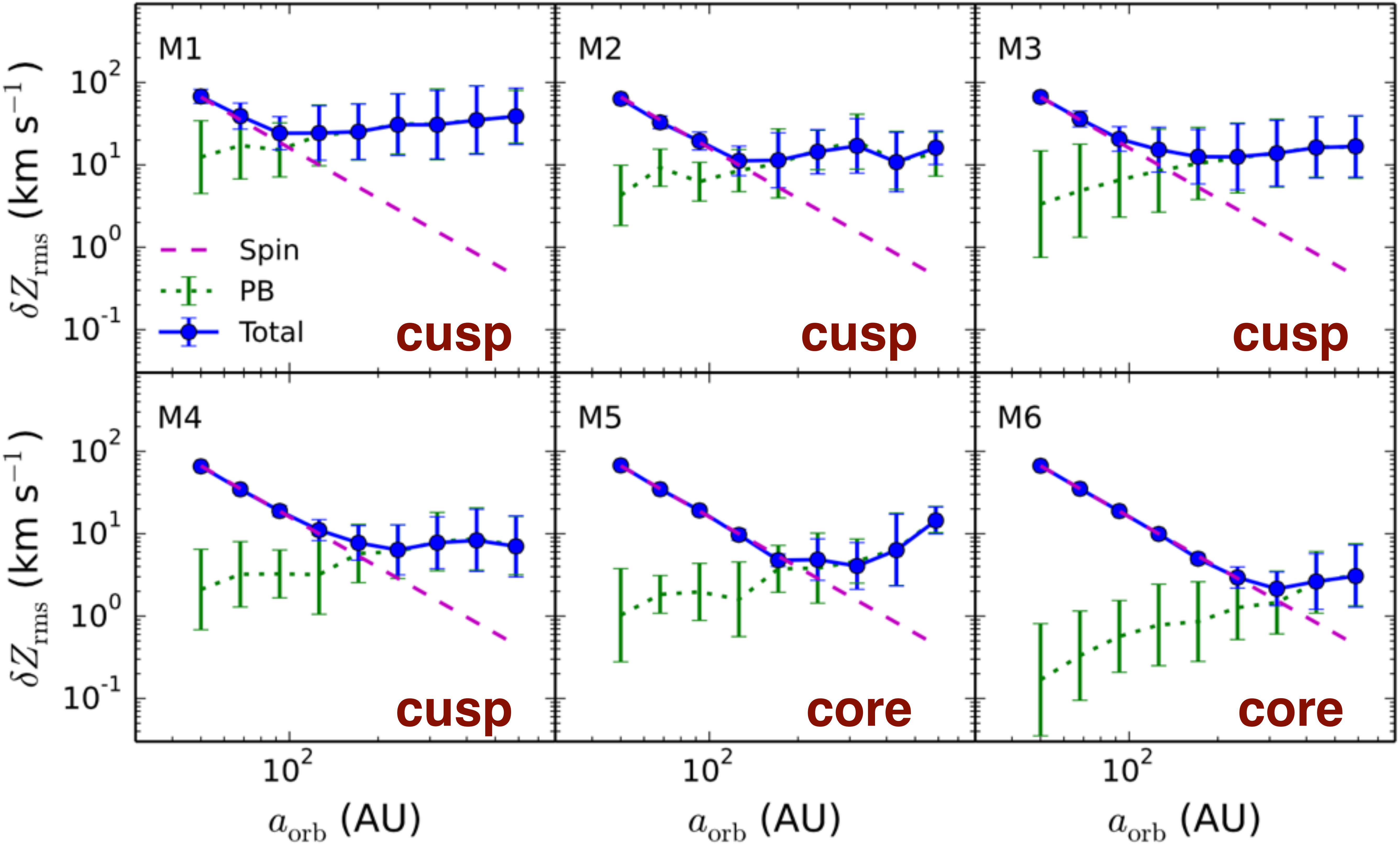
$$f(a_{\rm orbp}) \propto a_{\rm orbp}^{2-\gamma} \qquad f(e_{\rm orbp}^2) \propto (1 - e_{\rm orbp}^2)^{-\beta} \qquad \beta \le \gamma - 1/2$$

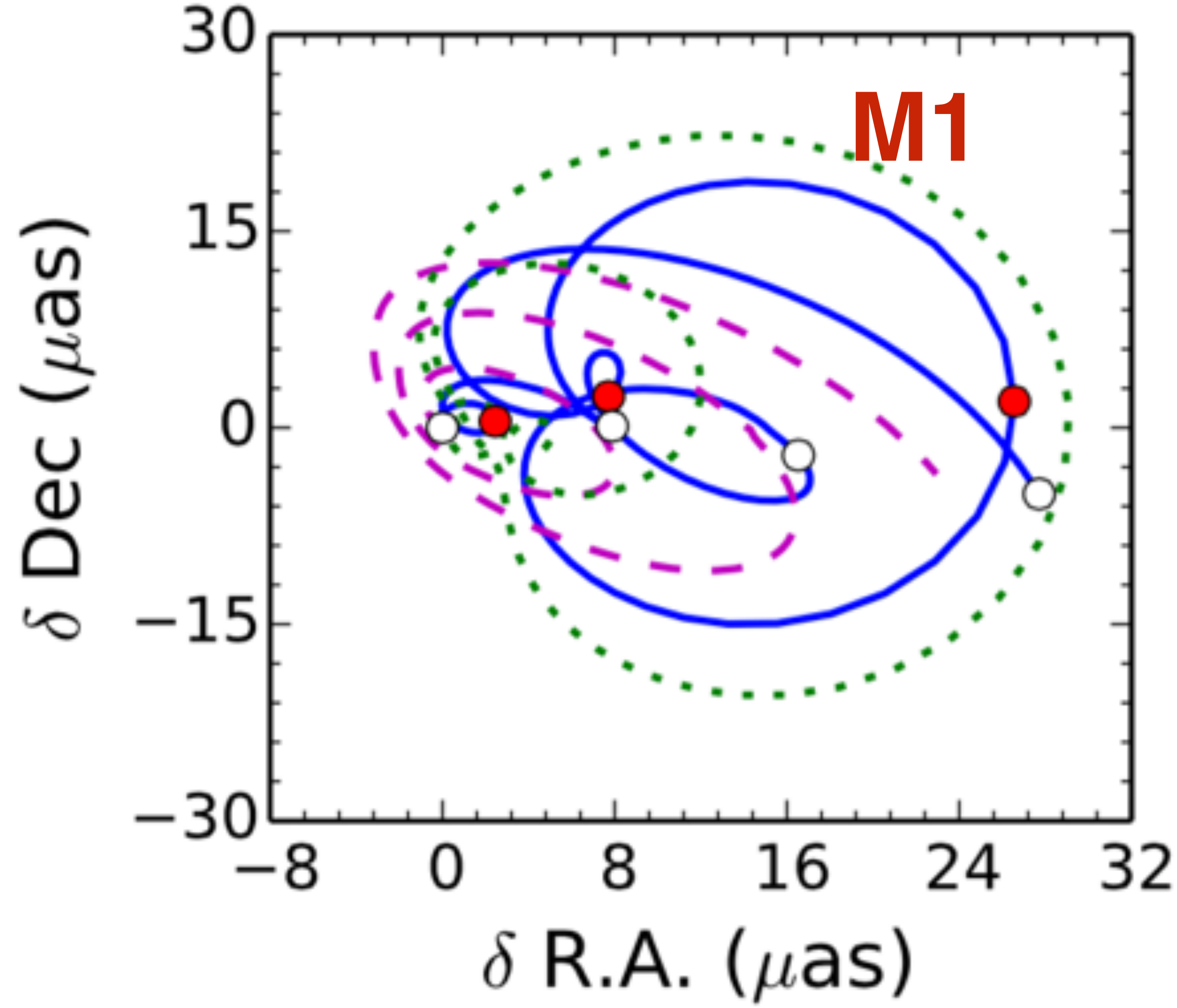
name	M_{10}[a] ($M_\odot$)	M_1[b] ($M_\odot$)	γ	β	m ($M_\odot$)	N_p[c]	$N_{\rm cluster}$
M1	1780	100	1.75	0.5	10	178	80
M2	1780	100	1.75	0.5	1	1780	8
M3	530	30	1.75	0.5	10	53	280
M4	530	30	1.75	0.5	1	530	28
M5	1581	5	0.5	-0.5	1	1581	10
M6	316	1	0.5	-0.5	1	316	48

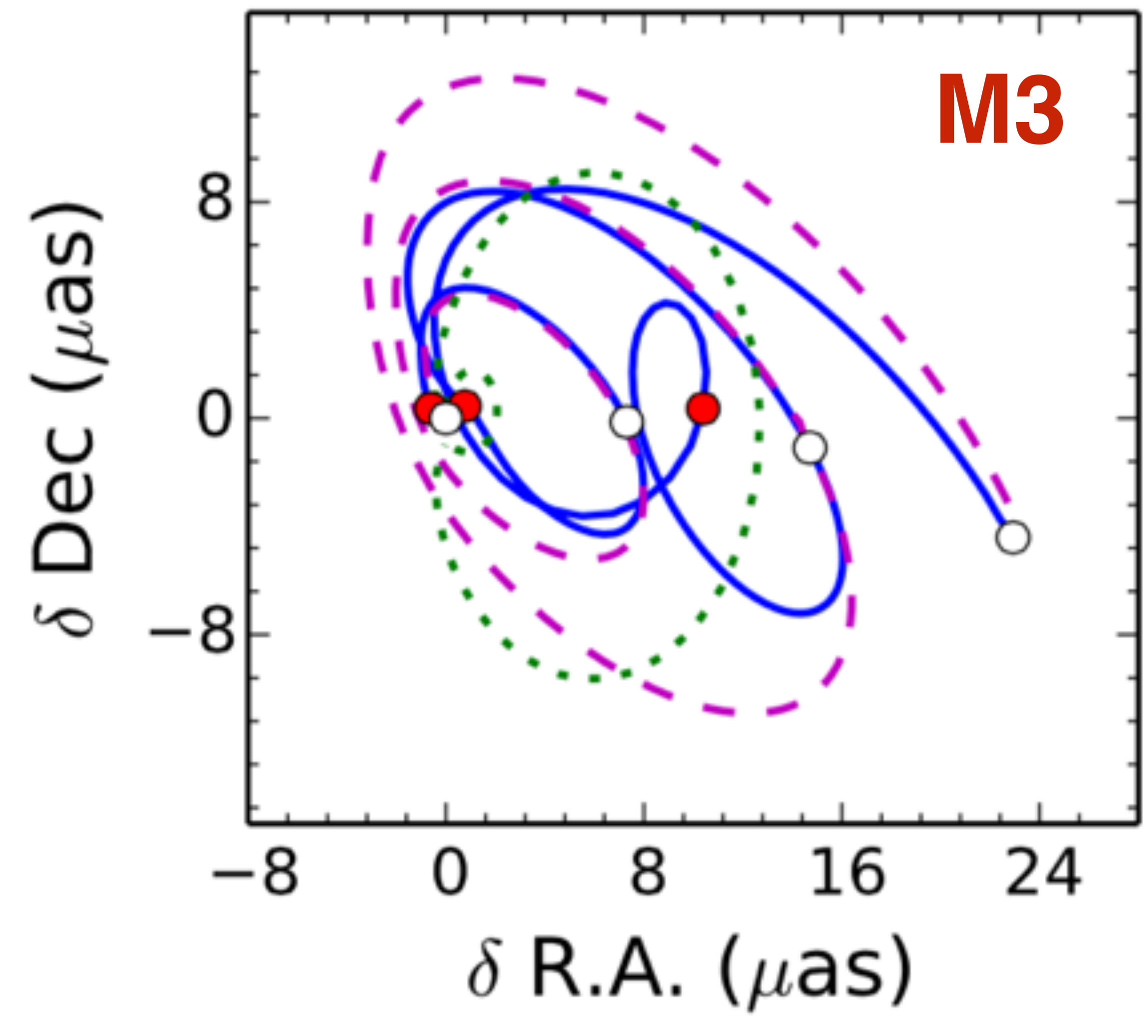
[a] The total mass of the stars with $a_{\rm orb} < 10$mpc($\sim$ 2062 AU or $\sim$ 0.26 mas).
[b] The total mass of the stars with $a_{\rm orb} < 1$mpc ($\sim$ 206 AU or $\sim$ 0.026 mas).
[c] The total number of the stars with $a_{\rm orb} < 10$mpc.

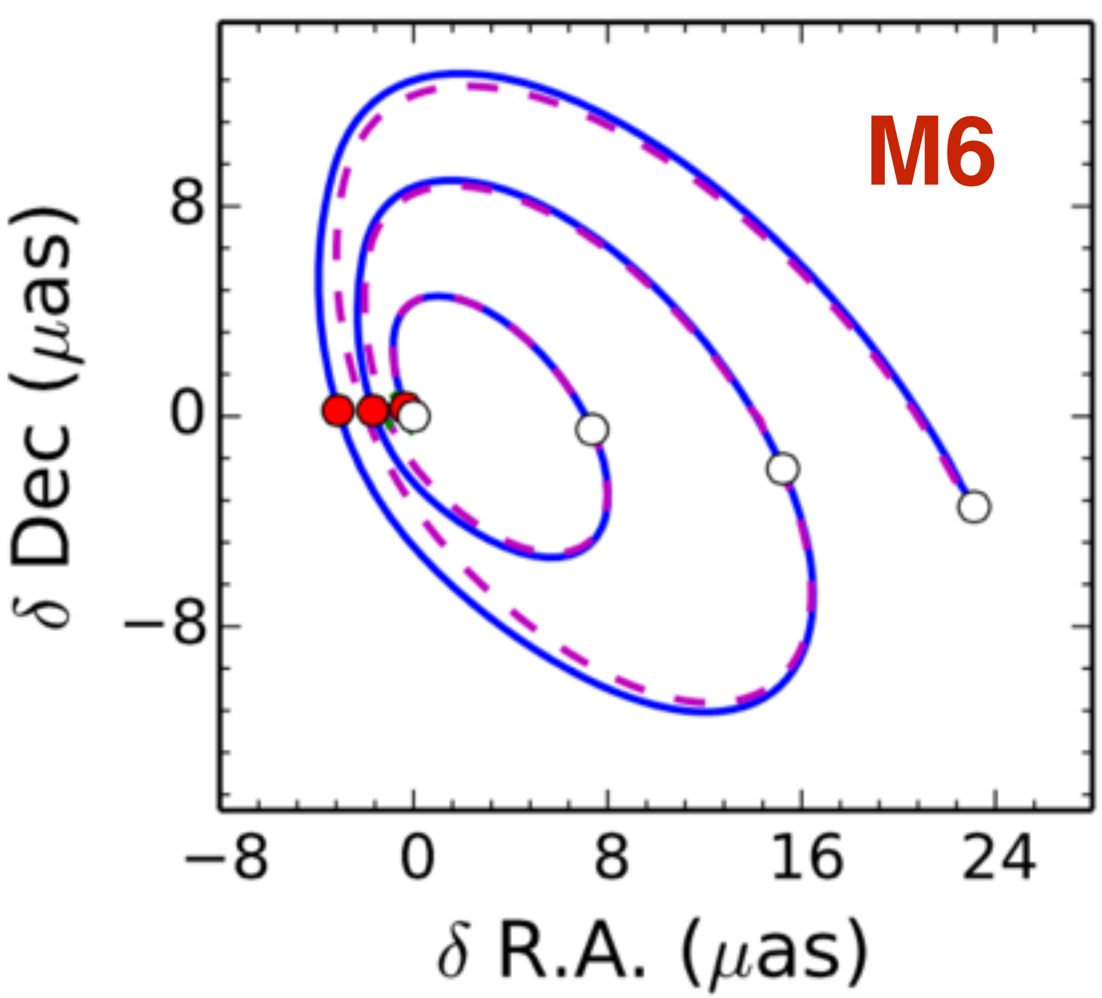
Stellar perturbation due to a star cluster




M1
M2
M3
M4
M5
M6
Spin
PB
Total
cusp
cusp
cusp
cusp
core
core
$\delta Z_{\rm rms}$ (km s^{-1})
$a_{\rm orb}$ (AU)
10^{2}
10^{1}
10^{0}
10^{-1}

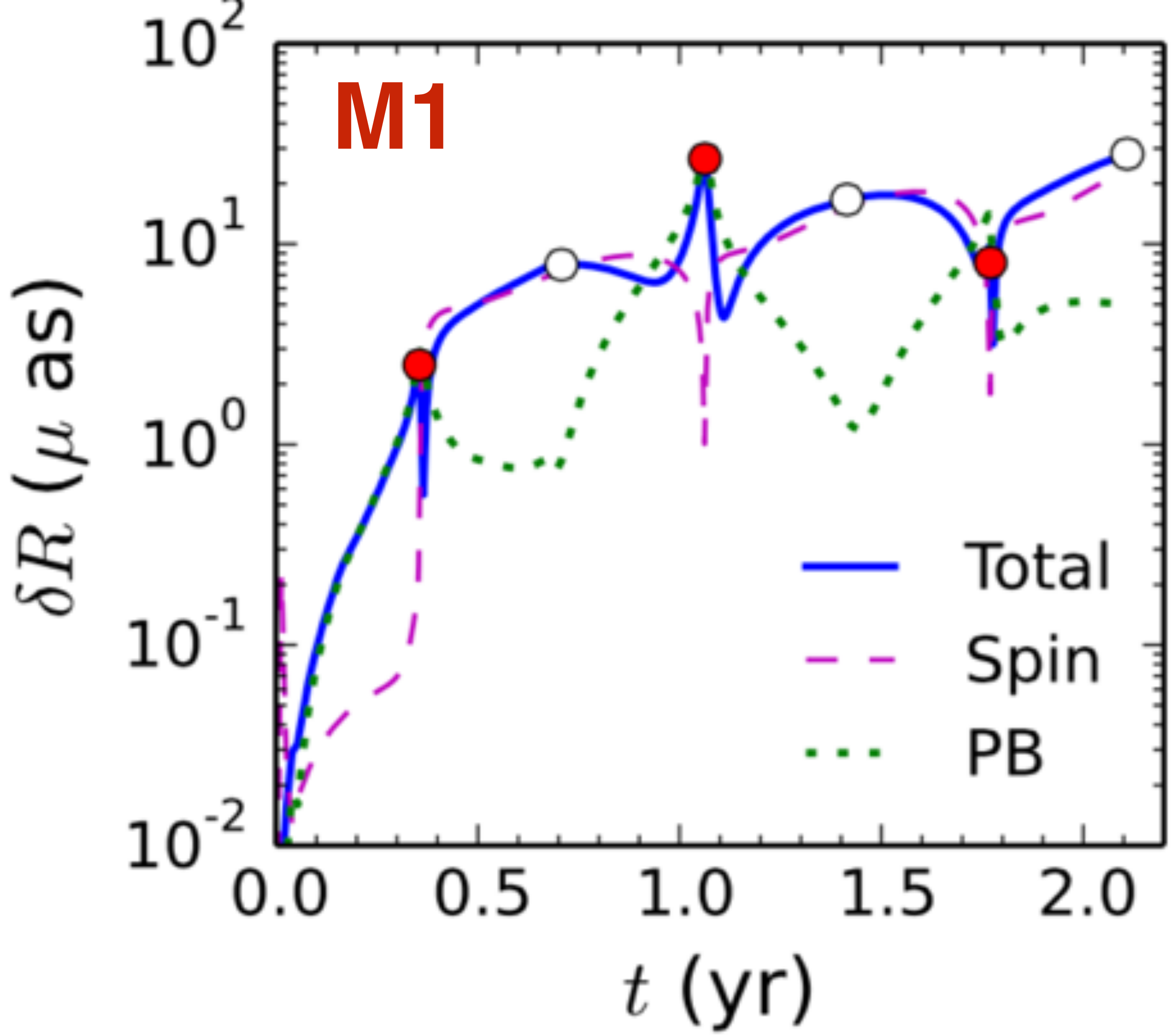


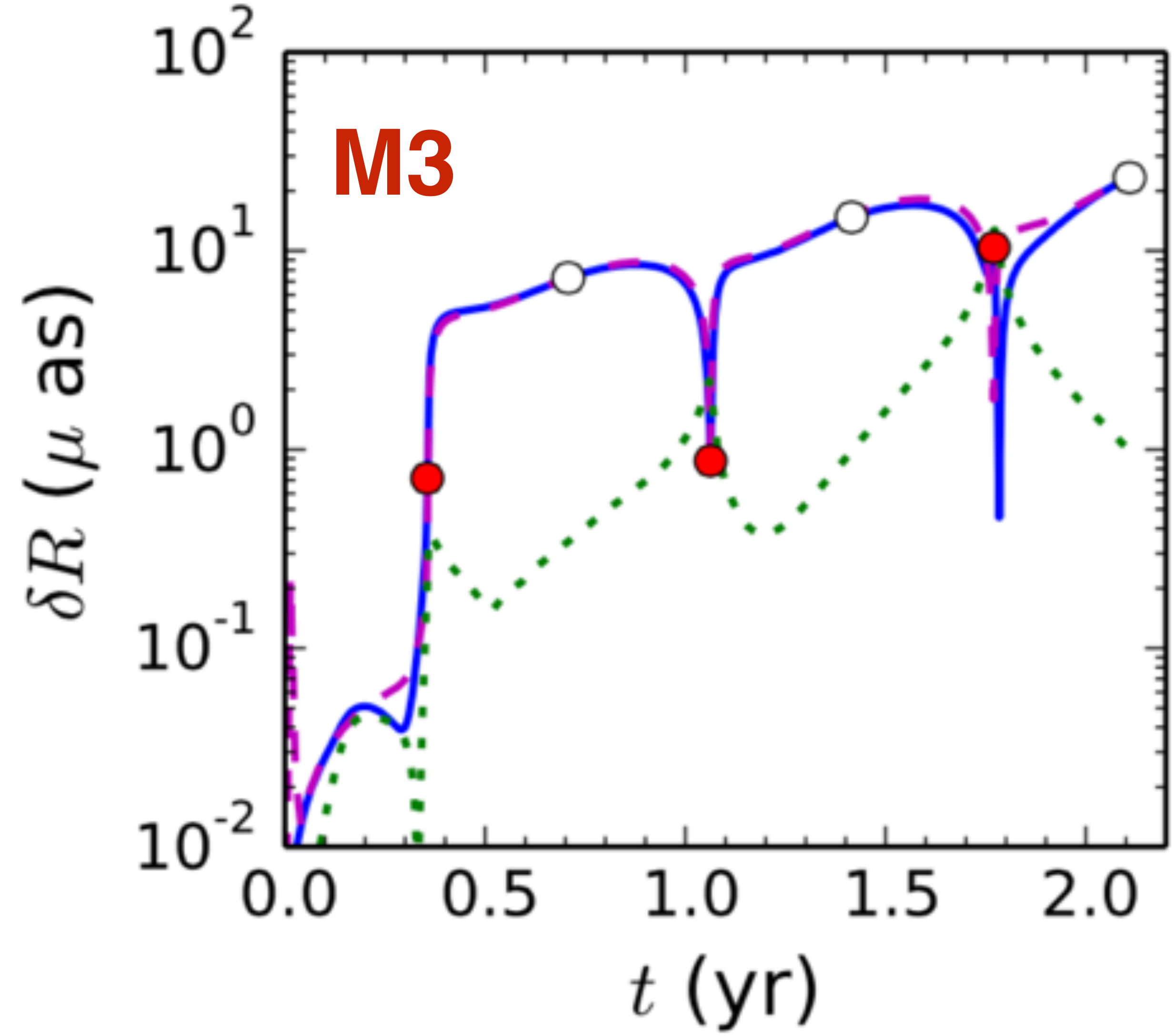


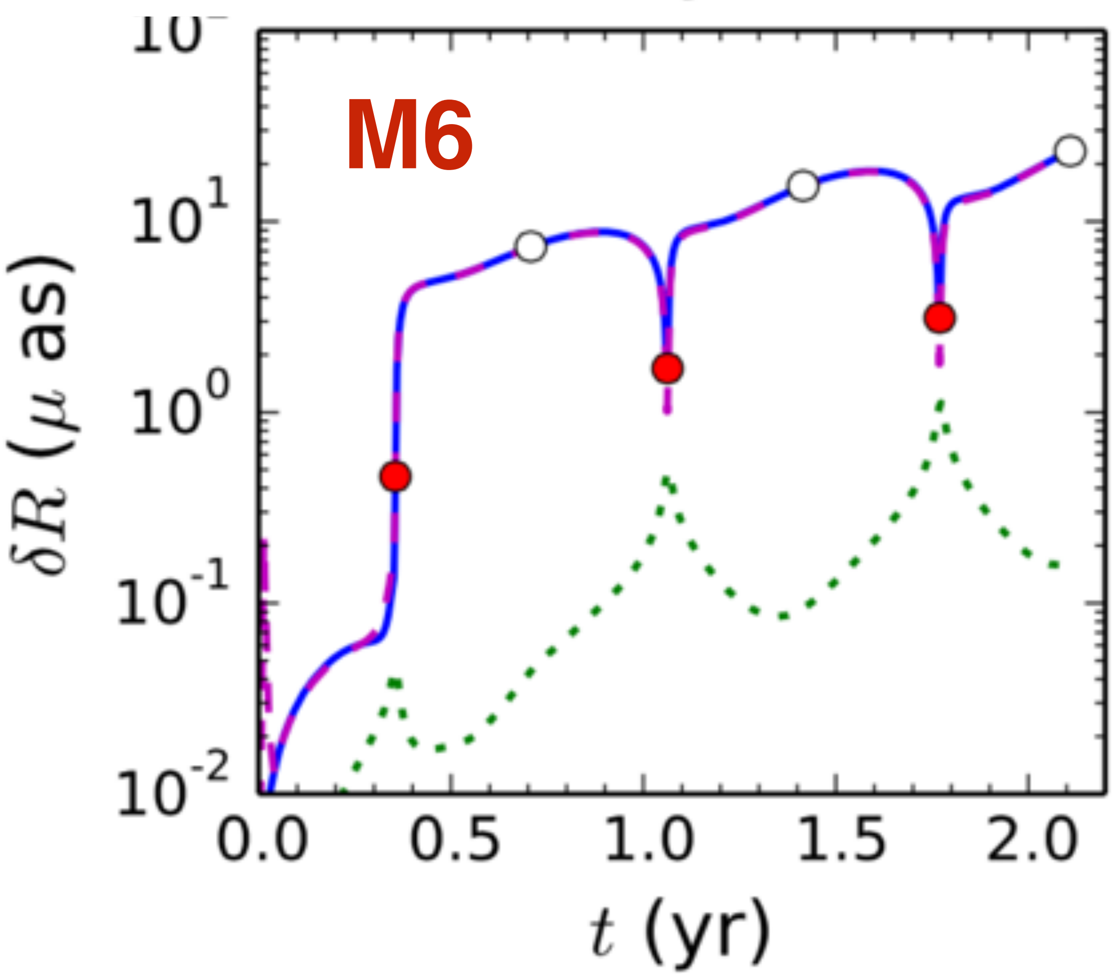


Position difference in sky position

Target star
a_{orb}=126AU

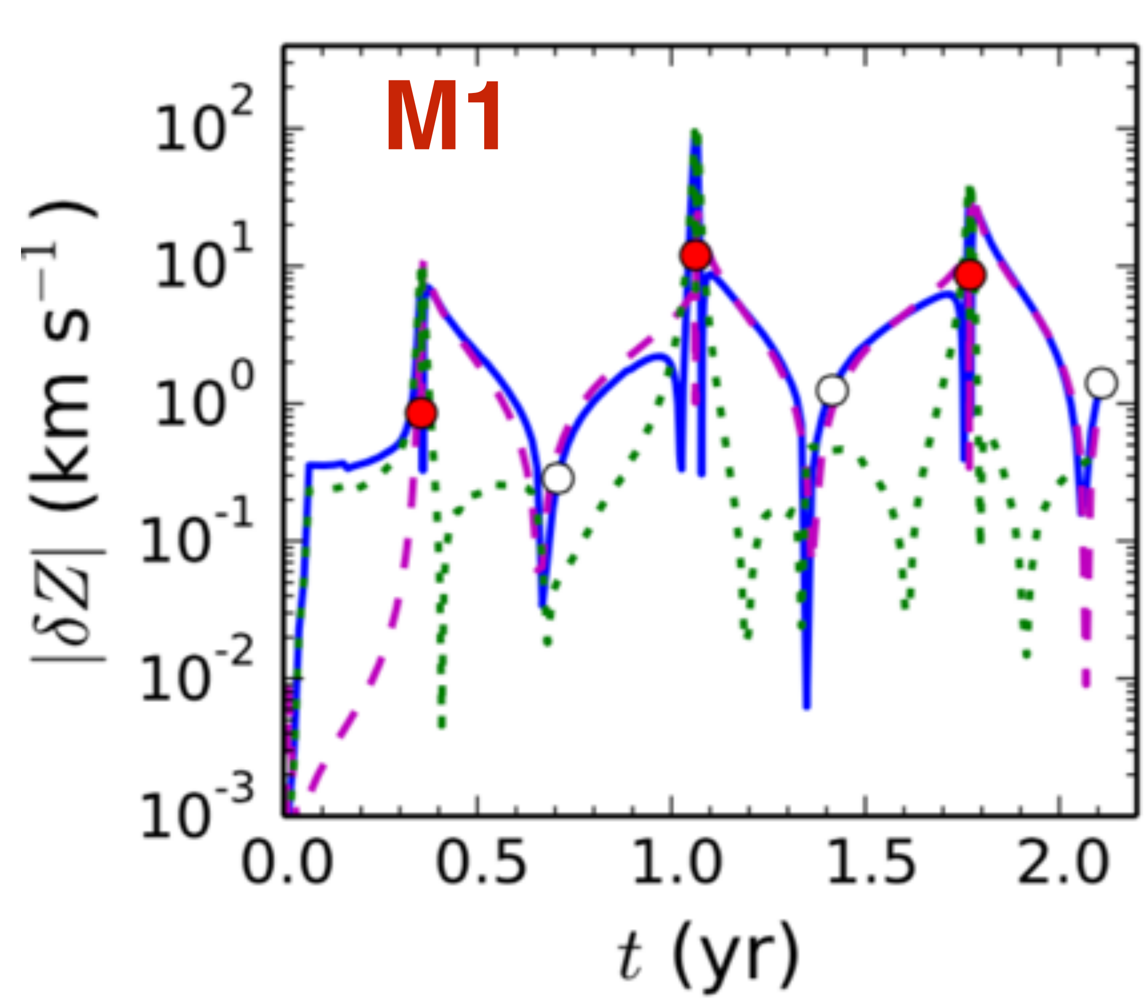


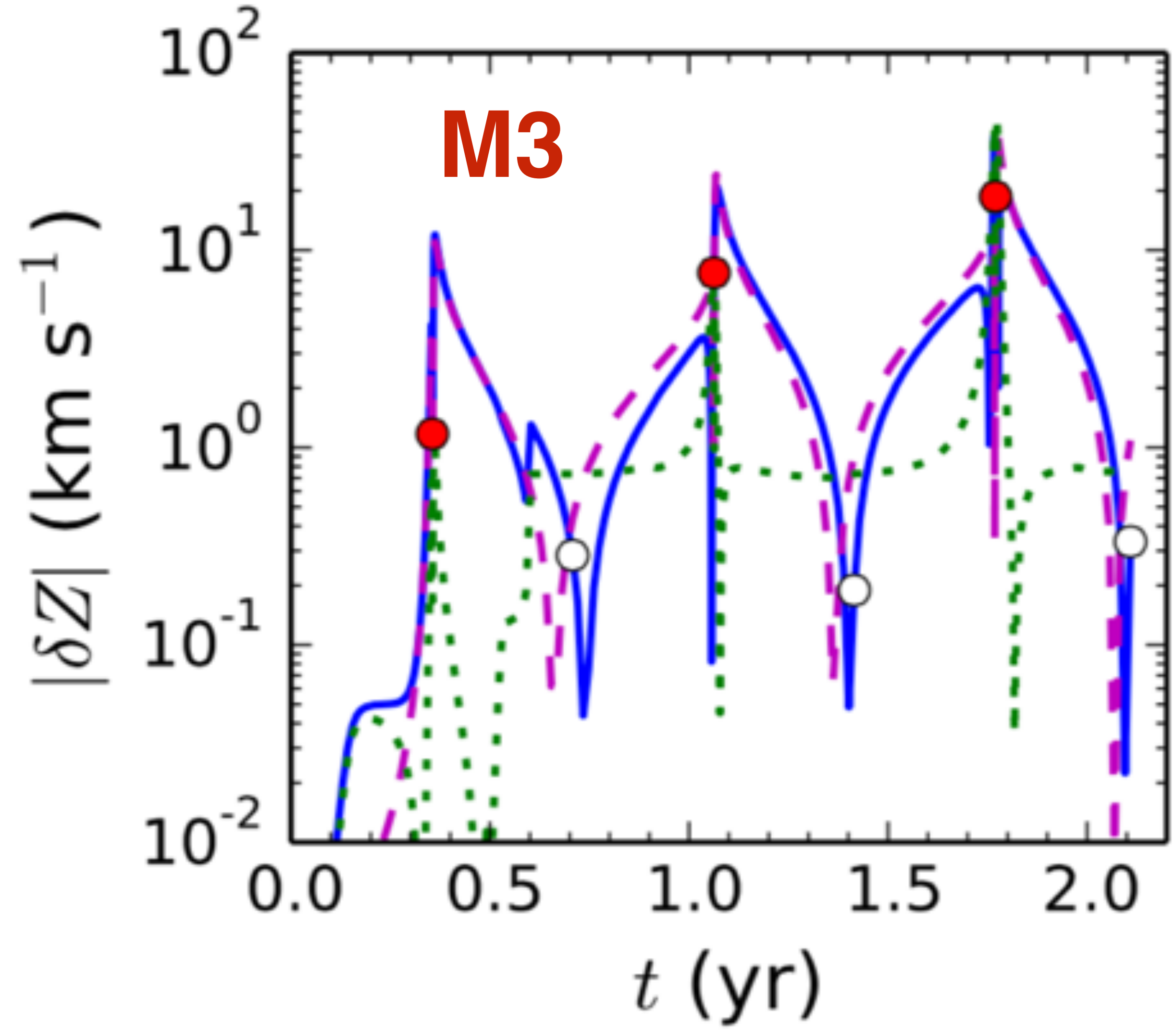


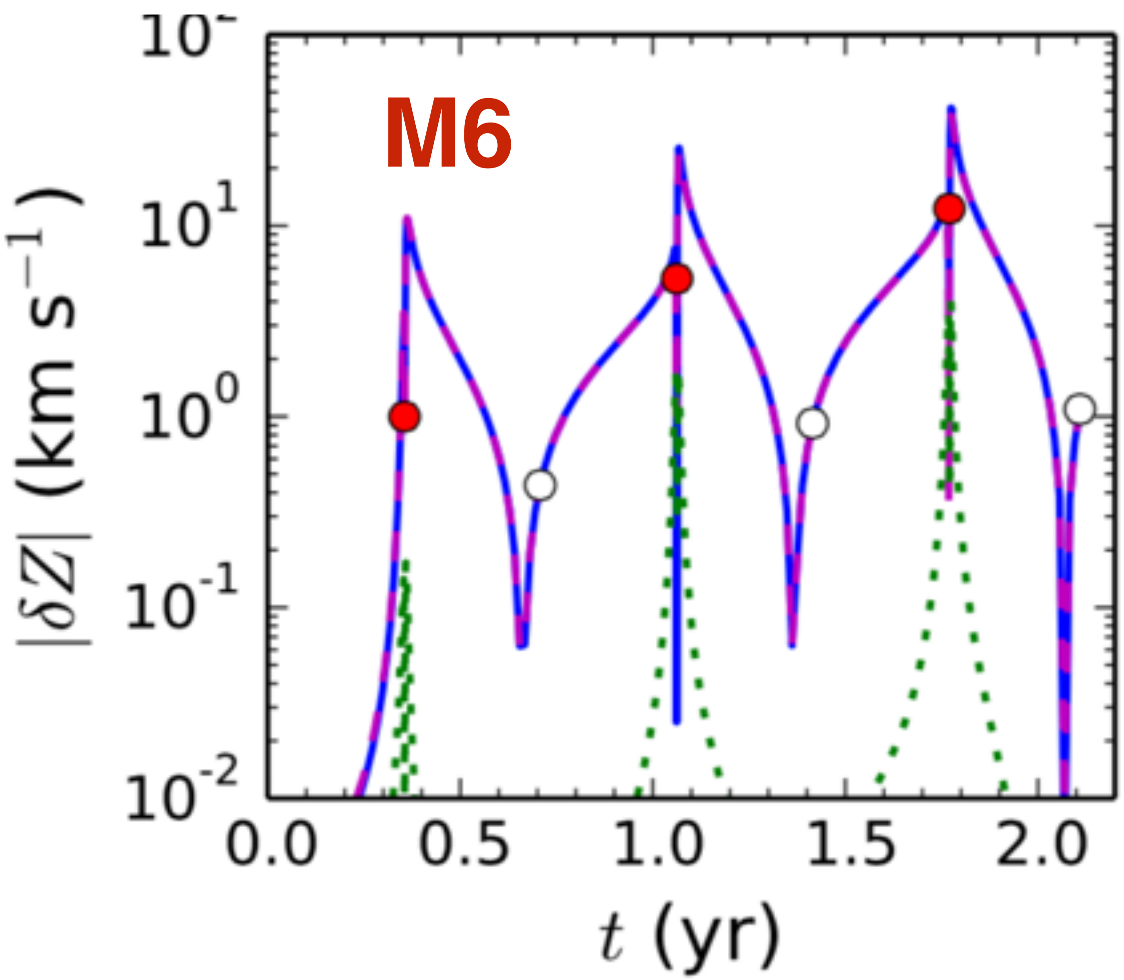


Position difference in sky plane

Target star
a_{orb}=126AU







Redshift difference

Target star
a_{orb}=126AU

Summary and discussion

- The spin-induced effects of S2/S0-2 are very likely obscured by the stellar perturbations from the S0-102.
- The stellar perturbations are dominated by perturbers inside the target star
- The stellar perturbations peaks around pericenter.
- Perturbed orbital period of stars
- The spin-induced effects dominates the signal for target stars inside 100-200AU if a clusters of stars exists around the MBH. But in principle the stellar perturbations are separable

Thank you!~~