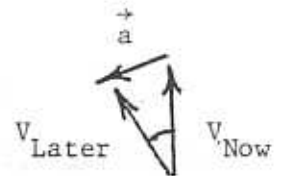
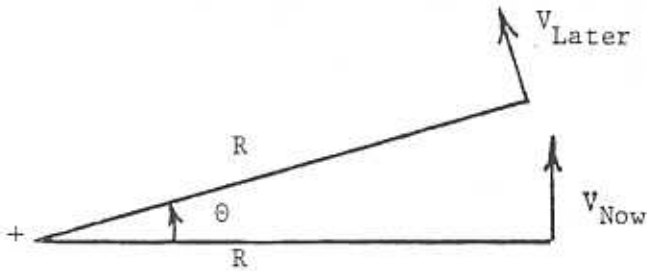
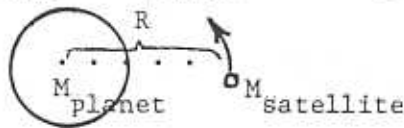


ORBIT: Uniformly accelerated circular motion under central force of gravity.
 Speed is constant, but velocity (a vector) is changing direction.

Center of Motion and Gravity



Acceleration is ΔV : vector subtraction from trigonometry,
 $a_{\text{acceleration}} = \Delta V = V \sin \theta$ small angle is approximately: $\sin \theta \approx \theta \approx V/R$
 so $a = V \cdot \frac{V}{R} = V^2/R$ and it points toward the center of the circular motion
 since $V = \frac{\text{Distance}}{\text{Time}} = \frac{2\pi R}{\text{Period}}$; $a = \left(\frac{2\pi R}{P}\right)^2 / R = \frac{4\pi^2 R}{P^2}$.



Newton's Second Law says: Force on satellite = $M_{\text{satellite}} \times a_{\text{satellite}}$

$$= M_{\text{sat}} \frac{4\pi^2 R}{P^2}$$

Use Kepler's Third Law:

$$P^2 = CR^3$$

$$= M_{\text{sat}} \frac{4\pi^2 R}{CR^3} \propto \frac{M_{\text{sat}}}{R^2}$$

So Kepler's Law tells us gravity is an inverse-square law force! By the same argument, and Newton's Third Law, the equal and opposite force exerted by the satellite on the planet must be proportional to M_{Planet}/R^2 .

The simplest expression for the force between satellite and planet is therefore:

$$F = G \frac{M_{\text{sat}} M_{\text{Planet}}}{R^2}, \text{ Points between the two centers of mass.}$$

↙
 Some
 constant
 of gravity

This is Newton's Universal Law of Gravitation.

Example: Newton's most famous use of Kepler's Third Law.

Consider an orbit close to the Earth's surface. The acceleration due to Earth's gravity, which we could measure by watching an apple drop, is 32 ft/sec/sec:

$$a = 32 \text{ ft/sec/sec} = 4\pi^2 \frac{R}{p^2}$$

R is the Earth's radius, equal to 4000 miles, so:

$$p_{\text{period}}^2 = \frac{4\pi^2 \cdot 4000 \text{ miles} \times 5000 \text{ ft/mile}}{32 \text{ ft/sec/sec}} = 25 \text{ million sec}^2, \text{ so:}$$

$$p_{\text{period}} = \sqrt{25 \text{ million}} = 5,000 \text{ seconds}$$

Thus, it takes any satellite in low orbit about 1 1/2 hours to circle the Earth.

Newton realized the Moon also orbits the Earth, but at a much larger distance, 60 times larger than the Earth's radius, so by Kepler's Third Law:

$$\begin{aligned} \frac{p_{\text{Moon}}^2}{p_{\text{Low Orbit}}^2} &= (60)^3, \text{ i.e.,} & p_{\text{Moon}} &= p_{\text{Low Orbit}} \times \sqrt{60 \times 3600} \\ & & &= 1 \frac{1}{2} \text{ hr} \times 8 \times 60 \\ & & &= 720 \text{ hours} = \text{one month!} \end{aligned}$$

That is Newton's famous synthesis, showing that the same gravitational force that makes apples fall also pulls the moon around the Earth in its nearly circular orbit.